Cygnus Loop nebula

Image credit: NASA/JPL-Caltech

Modern Cosmology for Everyone

Origin, Evolution and Fate of the Universe

RESEARCH PROJECT

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1. INTRODUCTION

1.1. Motivation

My motivation to have modern cosmology as the topic of my research project came for many reasons.

First, I am very interested in science and especially, in physics. From the wide range of branches of physics, I chose cosmology because it is a pioneering science that tries to understand the origins, the evolution, the properties and the end of our universe. Therefore, for me, it is the unification of many research fields in physics. It uses many principals of special and general relativity, classical mechanics, particle physics, thermodynamics, etc., in order to create a general and accurate picture of the cosmos.

In addition, cosmology also unifies two aspects of science: theory and observations. On the one hand, cosmology can be very theoretical. Models and equations are proposed based on previous knowledge and assumptions about what is studied. On the other hand, those models have to be tested with observations or the observations are simply used to create new models. This duality of the subject makes me enjoy it even more, since it is diverse and challenging.

Another reason why I chose cosmology as my topic for this project was that I got introduced to it during a practicum at the *Centro de Astrofísica da Universidade do Porto* (CAUP) in July 2014 with the *Joves i Ciència* program. There I also got introduced into the dark energy research on which my practical part is going to be based on.

But my motivation to do this work was not only based on my love for the subject. It was also based on the urge of explaining and reasoning those complex ideas to the world. Good work has already been done by exposing concepts like the acceleration of the Universe or the inflationary theory to the public. However, people that don't have a scientific background have to trust on what scientists are telling them. Therefore, I will try to prove some of the latest discoveries in cosmology and provide a detailed and as simple as possible reasoning of those concepts.

This idea originated basically because some of my friends did not believe in concepts like the expansion of the Universe. Of course their scepticism is justified, since they have only heard about it on TV or have only seen it on the internet. They would have to look for various sources and maybe wouldn't even understand them. So, I thought I could make a piece of work that could try to make some ideas of science more understandable and reasonable.

Furthermore, modern cosmology is quite compatible with this kind of work, since it tries to answer the questions that have been asked for millennia. These answers shouldn't be hidden from the broad public.

1.2. Objectives

The main objective of this project is to provide a general exposition of the area of modern cosmology, while trying to have something for almost all kinds of lectors. Usually, everything will be explained for the broad public by giving a more detailed explanation than usual in the scientific world, but some bits will require some scientific experience. However, those bits are not essential to the general understanding of a concept, even though they allow us to have a deeper knowledge of it.

Another objective is showing how current cosmological studies are done by proving the expansion and the acceleration of the Universe, showing possible outcomes of the universe as well as possible origins, showing the possible nature of dark energy and the prove of a diversity of cosmological models.

1.3. Questions or hypothesis

Some questions we could ask are:

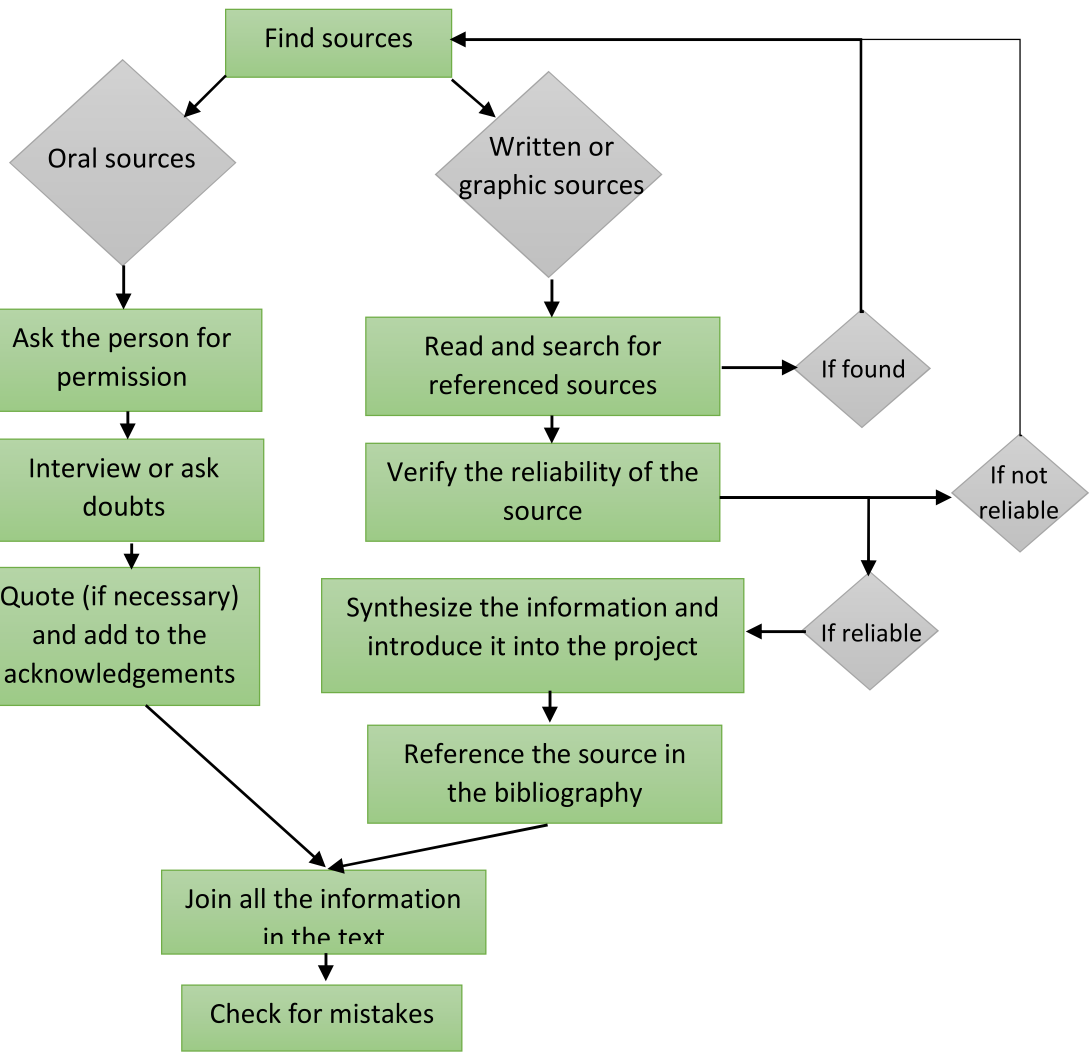
- What is modern cosmology? Does it affect me?
- How does a modern cosmologist work?
- Is our universe expanding? Is it accelerating?
- What makes our universe accelerate? What is it?
- What are the origins of our universe? How could its end be?

1.4. An approach to the methodology of the project

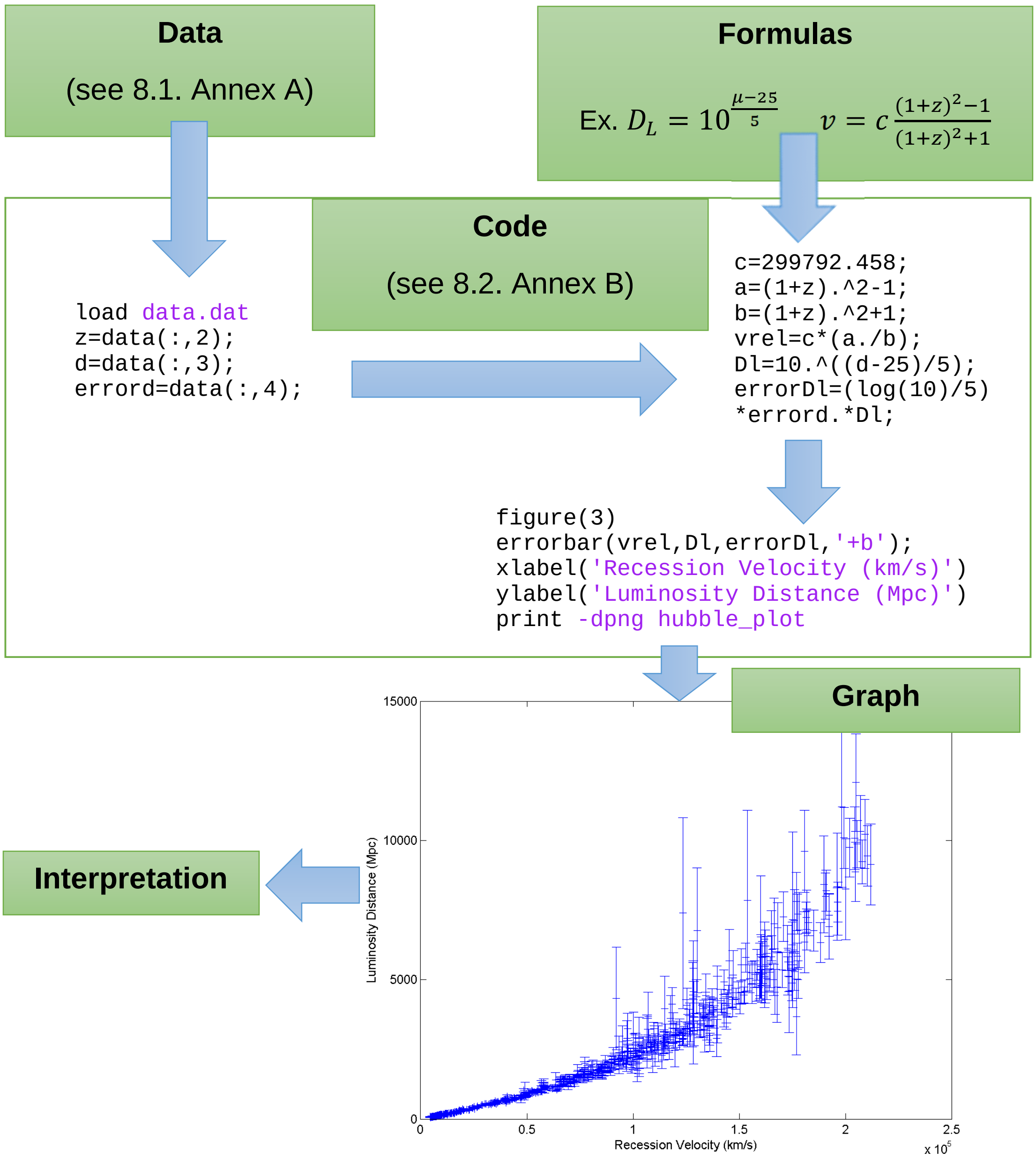
In order to make the practical part, two different types of sources were used: written sources and oral sources. Most used sources are written. First, they had to be found and read. After testing their reliability, the information was synthesized. Then, a bibliographical reference was created. The format of the reference depends on the type of source (book, article, website…). All the images were obtained from the Internet.

Oral sources were only consulted to solve doubts. Various scientists were contacted, for example, Prof. Dr. Carlos Martins.

Finally, the information was gathered and revised. The project was read multiple times and checked by the tutors. The whole process is described again by the following algorithm:



The methodology for the practical part consisted of writing a computer code in the language of MatLab that regulated an algorithm that statistically analyses the used data tables (see 8.1. Annex A). The code repeatedly solves formulas using hundreds of different values. Then, the code can graph these values. Finally, these graphs can be interpreted in order to see the physical phenomena that hide behind it. The process is explained in detail in the practical part of this project and in chapter 8.2, and it can also be seen in the following diagram:



The whole project has been translated from English to Catalan. It was first written in English, because most sources were in English, since it is the international language of science.

1.5. Materials and resources

- Computer (with a high computing capacity)
- Software
  - MATLAB R2014a: to make calculations and generate graphs.
  - Microsoft Office 2013: to write the text and the formulas.

1.6. Field of Study and Context: subject, time and space

This project is found inside the field of physics. Then the project specialized in the branch of modern cosmology. Modern cosmology is called modern, because conventional cosmology in its origins is more intensively related to philosophy than to physics and science. This is due to the fact that not until recently we had the technological ability to do precise enough observations or even to do complex calculations.

Today the area of cosmology is at its peak. It has more resources than ever to do observations (with telescopes like ALMA, Hubble, ELT, etc.) and to do calculations, like many supercomputers. In addition, recent advancements in particle physics or quantum physics allow scientists to create more accurate cosmological models. Nevertheless, right now there are so many topics to research on in cosmology that the available resources are not enough to satisfy the demand.

2. THEORETICAL PART

2.1. Origins of Cosmology

Cosmology is the study of the Universe and its components. Mainly it studies its origins, its evolution and its future. Nevertheless, today some people make a distinction between cosmology and cosmogony. Where cosmology studies the evolution, the properties and the destiny of the Universe and cosmogony studies only its origins. However, for simplicity, when referring to cosmology the first definition is meant. Modern cosmology studies those aspects using the scientific method and modern technology.

This wasn't always like that. In the beginning cosmology was based on religious and philosophical ideas about the workings of the Universe. Therefore, cosmology was a "science" that humans practised since the first time somebody wondered about his surroundings and how it works. First, it was purely religious, but, as some cultures advanced in astronomy and other sciences, cosmology became closer and closer to a science.

The first records of practices that could be called cosmology appear during the Neolithic. There, between 20,000 and 100,000 years ago, local groups tried to understand the processes around them. For them the Universe only was their closer environment. They especially focused on the weather, earthquakes and other events that they considered cosmological.

The word "cosmology" comes from the Greek words *kosmos* (world) and *–logia* (study of). From there originated the modern Latin word *cosmologia*. The English term "cosmology" came from the French word *cosmologie* and was first used in the 17th century.

2.2. Evolution of Cosmology

2.2.1. History of Cosmology

The earliest signs of cosmological thinking appear at about 20,000 BC when what appears to be a lunar calendar was made on a bone fragment. Another expression of cosmological ideas where the megalithic structures like Stonehenge that appeared in Europe and Africa during the Neolithic. However, one thing that was true for all cultures of that period is that they all had an anthropomorphic or personified view of the world and that they expressed those believes by spending a lot of time and resources in the making of monuments or artefacts.

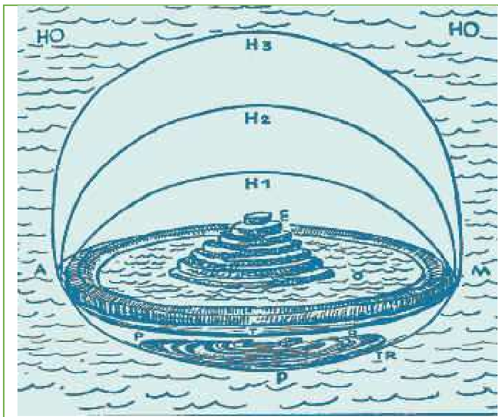


Image 1. Representation of the Babylonian universe. Source: *abyss.uoregon.edu*

At about 5,000 BC the first cultures appeared. Between the rivers Tigris and Euphrates, in today's Iraq, Kuwait and parts of Syria and Turkey, the Mesopotamian culture appeared. This culture mainly believed in the **Babylonian cosmology**. They believed in a flat world floating in water which consisted of multiple layers. They differentiated between the surface, the underground of Abzu (god of the underground water) and the underworld (where the gods lived). In addition, they believed that the sky and heaven had multiple layers: the firmament with the stars, the Sun and the Moon, the "ocean of heaven" and the "heaven of heavens". The sky and the Earth were kept together in a dome held by the so-called "pillars of heaven" and "pillars of earth". In Babylonian cosmology, the whole dome of the world floated in the infinite "waters of chaos". Their creation theory stated that the god Marduk created the Earth on fresh water and was surrounded by salt water.

It's obvious that the Babylonians gave great importance to water, since their culture was based on the abundance of water thanks to the rivers Tigris and Euphrates. Therefore, they based their cosmological model on this resource and, even though they already made a lot of astronomical observations, they never created a cosmological model based on their observations.

During the same period the **Egyptian** culture appeared. Its cosmology was of a very practical nature. It appeared from the need to count time and make predictions about the seasons. Following these needs they developed a lunar calendar and a solar calendar. Then, in order to explain those astronomical phenomena they developed myths and gods assigned to certain celestial bodies, constellations or events. The most important god was Ra, the god of the Sun. This shows us that already for the Egyptians astronomical or cosmological bodies, like the Sun, were more important than terrestrial resources. They thought like this because they were the first to realize that the cosmological bodies really were governing the cycles of life and the terrestrial resources.

At about 2,000 BC, on the other side of the world, the **Hindu** culture's **cosmology** appeared. It is recollected in the *Rigveda*. The *Rigveda* is a sacred collection of hymns in the Indo-Aryan language Vedic Sanskrit. The still practised cosmology believes that our universe was existing in a multiverse and was destroyed and recreated every 311.4 trillion years. For the Hindu culture the Universe was created with a universal egg. The three main gods of Hindu religion and cosmology are *Brahma* (the creator), *Vishnu* (the maintainer) and *Shiva* (the destroyer). They believe that the Universe consists of a giant egg that was half full with water (Garbhodaka Ocean) and of another half with many worlds levitating in the middle.

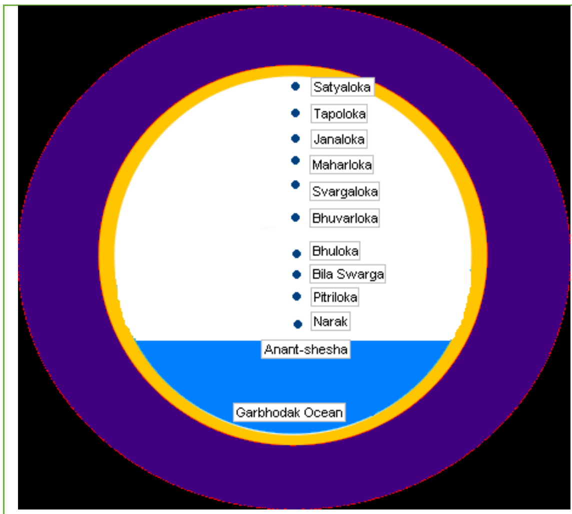


Image 2. Diagram of the Hindu universe showing the order of the different worlds. Source: *pparihar.com*

Hindu cosmology is very detailed and presents many variations. In addition, at the time of its formulation it was very modern, since they believed in multiverses. They thought that the Earth was a sphere and that it wasn't the centre of the Universe.

One of the cosmologies that most influenced the catholic and western culture was **Greek cosmology**. There were many different theories from many different authors that were created from 600 – 300 BC, but they all had some things in common. On the one hand, almost all of them were developed by philosophers that used an intellectual approach. On the other hand, some were based on mathematics and geometry. This gave birth to the modern scientific thinking that is applied to modern cosmology. The most remarkable theories in chronological order are:

- <u>Eleatic cosmology</u> (Parmenides of Elea):

  It states that the Universe is a finite sphere that doesn't change, doesn't have a beginning or an end and that the void doesn't exist. It also states that any change in the Universe is due to ignorance caused by experience from the physical world.

- <u>Atomist universe</u> (Leucippus and Democritus):

  This theory says that the cosmos is made of parts: an infinite number of atoms and an infinite extend of void. All atoms are made of the same substance, but they differ in size and shape. At first the force that drove the change of the atoms arrangements was randomness. Later, Leucippus stated that randomness is impossible and that every event in the Universe has some kind of purpose. They did not believe in the existence of gods.

- <u>Pythagorean universe</u> (Philolaus of Croton):

  It was based on the pre-Socratic astronomical system from Philolaus. This model was the first coherent system of the astronomical cycles. It consists of a great "Central Fire" orbited by the Earth, the Moon, the Counter-Earth, the other planets and the Sun. The Earth is revolving in a 24-hour cycle around the invisible Central Fire which covers the Counter-Earth. The Sun also orbits the Central Fire with a period of one year. Therefore, this theory is one of the first non-geocentric astronomical systems, but it isn't quite non-heliocentric.

Even though this model was developed by the Greek philosopher Philolaus, it wasn't named after him. Instead, it was named after Pythagoras, because Philolaus was his follower and a student at his school.

- <u>Stoic universe</u> (Stoic School):

  The stoic universe is a finite "material" inside an infinite void. For them this material is God or Nature and can be of two different kinds: matter or Fate. Matter is static and requires Fate to be changed.

- <u>Aristotelian universe</u> (Aristotle):

  This universe consists of a static sphere inside the not-existence. Inside, it is formed by four elements: earth, water, air and fire. Each body, depending on what it is made of, will have a different nature and a kind of movement associated to it. For Aristotle, some objects tend to stay still while others tend to move, but the most perfect movement is the circular motion. It is the most perfect because it governs the astronomical objects. In his theory, astronomical objects were made out of a perfect material which had the tendency to move in circles (the divine shape). Also, if circles are the most perfect shape for movement, he argued that the most perfect shape for bodies was the sphere. Therefore he accepted the Earth to be a sphere. His model was mathematically and geometrically proven by him with an array of 55 spheres to make his geocentric system fit to reality.

Image 3. Depiction of the Aristotelian universe made by Petrus Apianus in 1539. Source: *www.sirbacon.org*

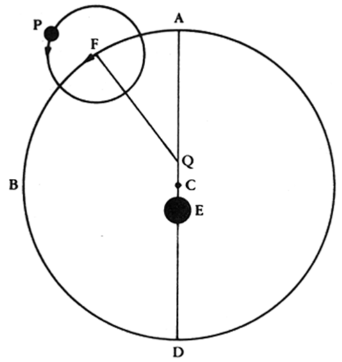


Image 4. Diagram of the Aristotelian universe with Ptolomenian updates. Source: *galileo.rice.edu*

In the second century AD, the Aristotelian system was updated by Ptolemy. He added another circular motion of the planets (epicycle, P), additional to their orbit (B) around the eccentric deferent (Q) that was next to the Earth (E).

This cosmological and astronomical model will be the most accepted for almost two thousand years in the western culture and by the Catholic Church, since for that period it was hardly questioned, because of its compatibility with the imprecise observations of the time was given.

- Aristarchean universe (Aristarchus of Samos):

  The Aristarchean universe was very similar to the Pythagorean universe. Only that Aristarchus replaced the “Central Fire” with the Sun. In that matter, he was the first person to propose a heliocentric model that places the Sun in the centre of our universe. In addition, Aristarchus thought that the stars might just be very far away suns.

In conclusion, almost every Greek theory (and also their predecessors) had a little bit of “truth” in them from today’s scientific perspective.

At about 500 BC, the Genesis creation narration was written. It is the base of creationism and **biblical cosmology,** but mainly cosmogony. It is the most accepted origins story by both Judaism and Christianity. However, it doesn’t talk much about its properties or fate. In biblical cosmology, our universe’s fate is described in the Book of Revelation. The Catholic Church in order to explain the properties of our universe mostly refers to Greek cosmology, for example, on the astronomical scale they accept the Aristotelian universe updated by Ptolemy.

The Book of Genesis is part of the Hebrew Bible and the Christian Old Testament. It describes the creation of the world by God in seven days. Each day was one step of the creation and on Sunday, God rested.

The Book of Revelation which is part of the New Testament explains the biblical apocalypse. It describes that there are seven seals and equestrians of apocalypse. After one seal is opened, one equestrian is unleashed and brings one of seven plagues to the world. When all seals are opened, there will be a final battle between Christ and the Antichrist with their armies. The Book of Revelation explains that Christ's army is going to win the war and only the truly good people will have survived the apocalypse to rebuild the cultures.

Medieval cosmology and philosophy didn't make a lot of progress from there. In the Western world the creationism and the Aristotelean geocentric model were still the most accepted. In the Islamic world a variant of creationism and the Aristotelean model were also accepted. Even though some heliocentric models were developed. During the Middle Ages the most important cosmological ideas were:

- Multiversal cosmology (Fakhr al-Din al Razi, 12th century):

  Fakhr developed a religious theory in which he stated that God had the power to fill the infinite void of space with as many universes and worlds as he wanted.

- Copernican universe (Nicolaus Copernicus, 15th century):

  The Copernican universe is the first heliocentric model that was widely accepted. At the time of its development by Nicolaus Copernicus the Catholic Church illegalized the theory and prosecuted all of its followers. However, on the long run, the theory was accepted since it fit in very well with observations and was simpler than the Aristotelean model.

  This cosmological model was one of the most important in science. Even though it is more astronomical than cosmological, it was a great step towards the Cosmological principle (see 2.3.1.b.), since it moved the centre of the Universe further away from us for the first time with success.

Pre-modern cosmology appears when the scientific method appears. The cosmology practised during that period of time was already very scientific and implied a lot of maths, but the technology and level of science at that time didn't allow to create as accurate models as the ones made later on. Also some models were still very philosophical.

One of the first true cosmological models was Bruno's cosmology (Giordano Bruno, 16th century). He believed that the Universe was infinite in space and infinite in time. Also he is one of the first to propose a non-hierarchical universe, a universe where the Earth and the Sun are nothing special in comparison to all the other astronomical bodies. His universe was also an atomist universe.

The Keplerian model (Giordanno Kepler, 16th-17th century) was another astronomical model that revolutionized the understanding of the motions in our Solar System. Kepler arrived to the conclusion that planets were not orbiting the Sun following spherical orbits, but that they were following elliptical orbits. This theory still holds true today.

A cosmological model that was accepted for a long period of time is the static Newtonian universe (Isaac Newton, 17th century). There Newton proposed a static and infinite universe in which matter was uniformly distributed, so the gravitational attraction balanced out. Nevertheless, later it became clear that this universe was too unstable and couldn't remain static.

After this long evolution of cosmology throughout the history of humanity, we finally arrive to **modern cosmology**. Modern cosmology, together with pre-modern cosmology, is the first cosmology that can be described as a science. It postulates a hypothesis that it tries to prove or disprove with scientific evidence. Modern cosmology is constantly evolving and it is still far away from creating one model that perfectly explains the entire Universe, but it tries.

Modern cosmology didn't really have a start, it appeared in a gradual process. However, here are some remarkable cosmological models of the last hundred years: *Einstein's universe* (Albert Einstein, 1917), *Friedmann's universe with spherical space* (Alexander Friedmann, 1922) or *with hyperbolic space* (1924), *Big Bang Theory* (Georges Lemaître, 1927-1929), *Friedmann-Lemaître-Robertson-Walker models* (Howard Robertson and Arthur Walker, 1935) and *Cosmic Inflation* (Alan Guth, 1980). These models are not going to be explained here, since most of them are included in chapter 2.3.

2.2.2. Cosmology and Religion

Cosmology has always been closely related to religion. In its beginnings, cosmology and religion were almost indistinguishable. Over time they slowly separated and finally an independent science emerged.

When the first *Homo sapiens* wondered about their environment and how it worked, they started developing ideas to explain what was happening. These ideas could be called cosmological, but at the same time religious. On the one hand, they were ideas that tried to explain the origins, properties and fate of the "Universe". On the other hand, they were of a divine nature and involved gods and myths. Those cosmological questions and their answers are the foundations of the great religions we know today.

Cosmology and religion remained completely unseparated until the appearance of philosophy. Especially in Greek philosophy, theories appeared that were based on logical reasoning and were less divine. For example, the atomists came to the conclusion that everything had to be made up of tiny indivisible particles and they excluded the existence of gods. However, religion and gods were still an important theme in philosophic cosmology.

After a very long time of dependence between cosmology and religion, with the appearance of the scientific method, cosmology became completely independent from religion. Modern cosmology was born. From this moment, all serious cosmological ideas were a scientific hypothesis that had to be tested experimentally with astronomical observations.

Cosmology was still present in religion, but it didn't evolve anymore there. All new non-scientific cosmological theories weren't accepted by most people or they were forgotten after a short period of time. Nowadays, cosmology is intensely connected to physics. It tries to find the scientifically accurate mechanisms and laws that govern our cosmos by formulating theories, questioning them and proving them via observations or experiments.

Cosmology is one of the most important practises in human history. It is the expression of our curiosity, the foundation of our cultures and the science of everything.

2.3. Modern Cosmology

2.3.1. Foundations of Modern Cosmology

Modern cosmology, as any other science, follows the scientific method. Therefore, it states hypothesis and tries to prove them experimentally. But how do cosmological experiments look like?

Cosmological experiments can be very diverse, since cosmological models are affected by almost every discovery in physics. A particle physics or quantum mechanics experiment that deals with the smallest building blocks of our universe, can prove or disprove a theory that tries to describe the entire cosmos.

However, when those models are tested directly usually some direct measurements of phenomena of cosmological scales (see 2.3.1.a.) have to be made. So far the only information carriers we can measure are photons and neutrinos.

Photons are the particles associated to the electromagnetic force or rather the constituents of light. They show an interesting quantum mechanical effect that is called wave-particle duality. This means that a photon is a particle and a wave at the same time. Although the views on this are diverse, it can be imagined as a particle that has a certain probabilistic wave assigned to it and that when it is measured or observed decides where to be, depending on the probability for each point.

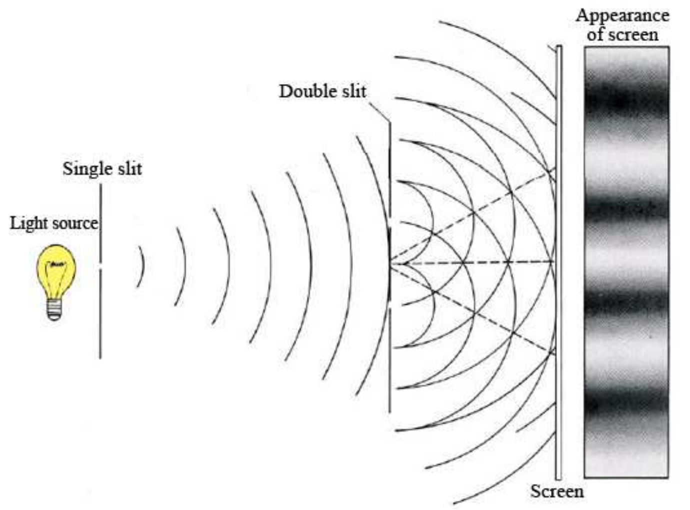


Image 5. Setup of the double-slit experiment. Source: *www.askamathematician.com*

This effect can be proven with a variation of the double-slit experiment by Thomas Young (see *Image 5*). Where implying that light is quantized (divided in indivisible units) and having a setup that can only sent one photon at a time through the double-slit and that guarantees that the photon won't be measured while passing the slit, a diffraction pattern like in *Image 6* can be obtained. It tells us that each photon independently "hits" the measurement panel at a certain point. However, when we look at all the photons, we can see that they create a pattern on the measurement panel that resembles the pattern of a wave-diffraction (see *Image 7*).

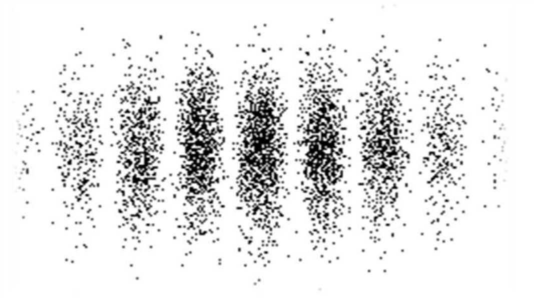


Image 6. Pattern produced by independent photons in the double-slit experiment due to wave-particle duality. Source: *media.tumblr.com*

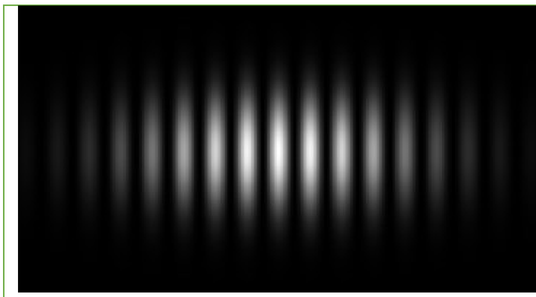


Image 7. Pattern produced by a wave in the double-slit experiment. Source: *www.tnw.tudelft.nl*

Photons or light can have different wavelengths. Each wavelength can have different causes and can be used to determine those causes. They are classified in wavebands and the most useful of them in cosmology are:

- Radio waves: They have very long wavelengths. They are useful for the detection of very far away galaxies.
- Microwaves: They have long wavelengths that in the cosmos almost exclusively form part of the Cosmic Microwave Background (CMB). This makes them one of the only information sources from the beginnings of our universe.
- Infrared: It is a wavelength that is very useful for the detection of young galaxies in our nearer galactic neighbourhood. In addition, it doesn't get absorbed too much by cosmic dust, which allows us to see through them.
- Visible light: It has the wavelength of light that our eyes can detect and our brain is able to process. Historically it was our only information source of the cosmos. It allows us to see the stellar bodies that are closest to us.

- X-rays: They have one of the shortest wavelengths that exists and they are essential to find galaxy clusters. With visible light, only the galaxies themselves are visible, but with X-rays, the X-ray emissions of the gas clouds inside the cluster get visible. Thereby confirming that this group of galaxies is a cluster.

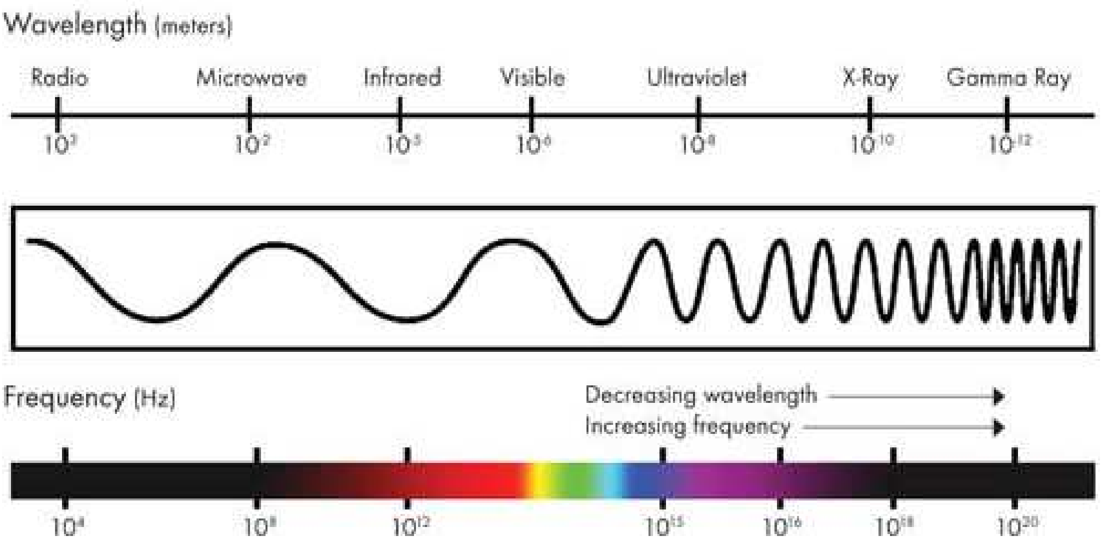


Image 8. Electromagnetic spectrum classified by wavelength and frequency. Source: *online.science.psu.edu*

Neutrinos are another information carrier that reaches us. They are particles moving at the speed of light that hardly interact with matter, since they don't carry charge. Thus, until recently, they haven't been detected.

They are created in certain nuclear reactions or radioactive decays. Since they can be created in stars, supernovae or even the Big Bang (see 2.3.4.b.), they are very important for cosmology. However, this gives place to another branch of cosmology, the neutrino cosmology, in which we won't get into.

a. Scales of the Universe

In science, when we try to describe certain phenomena, we move within a certain scale of the three-dimensional space. The same goes for cosmology. Cosmology mainly operates on the scale of the "entire" Universe, when we achieve large-scale smoothness. In order to understand this, *Image 9* provides a rundown from galactic scale to universal scale.

The concept of **large-scale smoothness** is the main component of the Cosmological Principle, the foundation of modern cosmology.

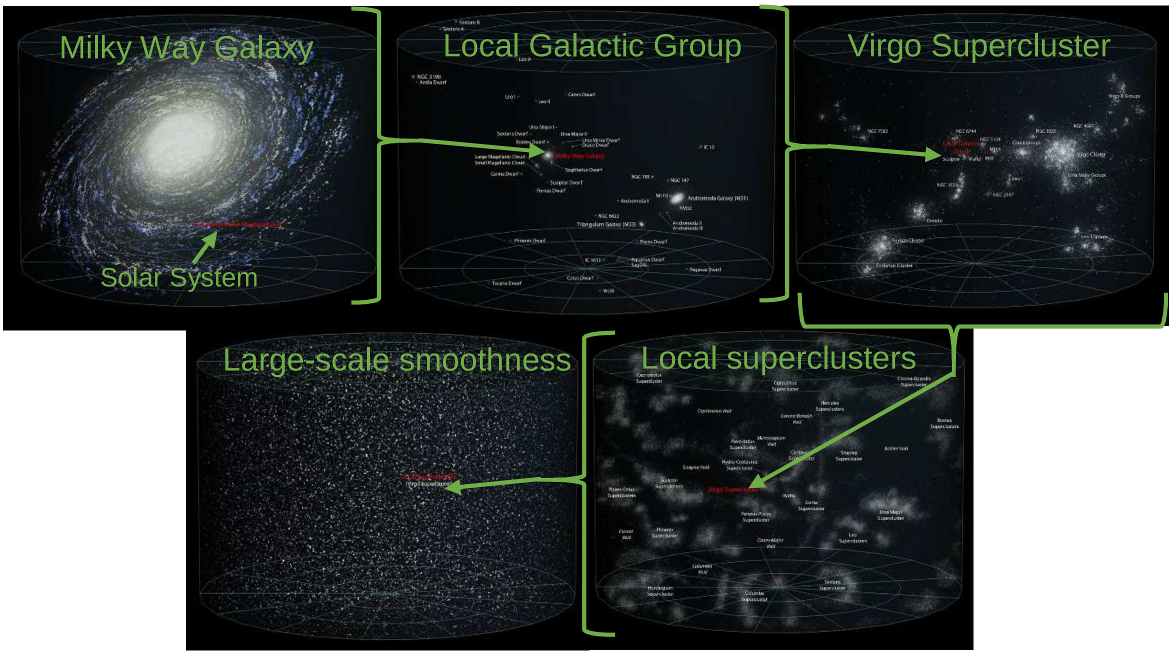


Image 9. Scale of the Universe from our Milky Way Galaxy to the large-scale smoothness. patrickmn.com

b. The Cosmological Principle

The **Cosmological Principle** states that, on a large enough scale, the matter distribution of the Universe is homogenous and isotropic. Homogenous means that the Universe looks the same at each point and isotropic means that it looks the same in every direction. This distinction is important since one doesn't imply the other.

Of course, the Universe doesn't present a perfect homogeneity and isotropy. Therefore, most current cosmological research tries to find different deviations from this principle and their causes.

c. Gravitation

In cosmology and in this project, we mostly use **Newtonian gravity**, since it still applies on large scales and simplifies the systems.

It is built on Newton's law of universal gravitation:

$$F = \frac{GMm}{d^2} \qquad (1)$$

Here Newton stated that the gravitational force a body experiences (*F*) is proportional to the product of the mass of the body (*m*), the mass of another body (*M*) - usually the one with greater mass – and Newton's gravitational constant (G = 6.67 x 10^{-11} $m^3kg^{-1}s^{-2}$). Also it is inversely proportional to the distance between the two bodies (*d*) squared. This means that if the distance between the two bodies (*d*) increases, the gravitational force will decrease with the square. Additionally, if the masses of the bodies increase, the force will increase and if they decrease, the force will decrease.

Newton's gravitational constant (*G*) is nothing else than the gravitational force two 1kg bodies would exert on each other if they were separated by exactly one meter.

Newton also described the gravitational potential energy with the following expression:

$$E_{Pg} = -\frac{GMm}{d} \qquad (2)$$

This expression causes gravitational potential energy (E_{Pg}) always to be negative, since mass or distance can't become negative. This represents the tendency bodies have to be close together, so they can only achieve "equilibrium" is they get closer to each other.

However, the Universe is a messy place and there are never two isolated masses that could exert a perfect force on each other without having some other attractive forces from other bodies. In Newton's **shell theorem** he proves that if we have symmetric matter distribution, the sum of the mass

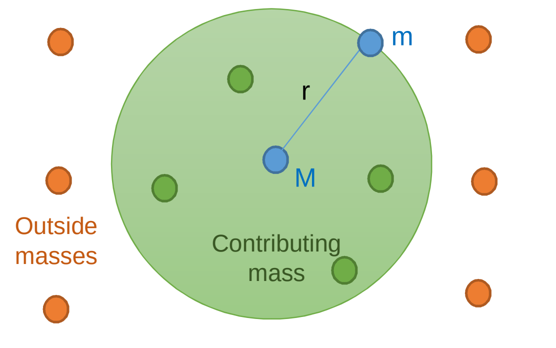


Image 10. Diagram of the shell theorem where M = Σ contributing mass and the outside masses cancel out. Source: Own

inside a sphere with a certain radius (*r*) can treated as if it were all accumulated in the centre of the sphere (*M*). The attractive forces on *m* from all the masses located outside the sphere eventually cancel out, since the force is the same from all directions.

This theorem only holds true for a uniform and symmetric matter distribution. Thus, it may not apply to small systems, but assuming large-scale homogeneity and isotropy it can apply to the Universe as a whole.

2.3.2. Expansion of the Universe

The **expansion of the Universe** is the metric expansion of space as time progresses. This means that every distant point in our universe is getting further apart from all the others. From Earth this looks like every distant galaxy is moving away from us while appear to stand still.

If you compare space to the surface of a balloon and distant galaxies to some dots on the balloon, if you now inflate the balloon, you will see that all the points are moving further away from each other. Just as the Universe is doing right now.

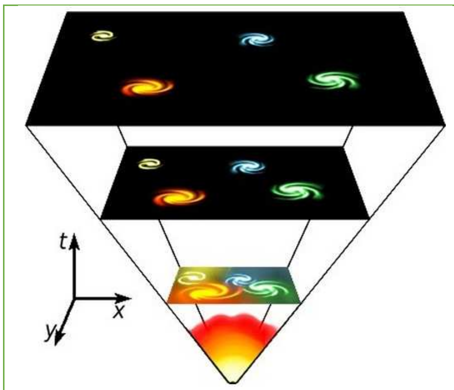


Image 11. Impression of the metric expansion of the universe as time progresses after the Big Bang. Source: *en.wikipedia.org*

The galaxies themselves or any other small bodies don't get bigger or get ripped apart because of this, since the gravitational attraction on this scale is way too strong.

This phenomenon is accredited to Edwin Hubble and is described by Hubble's Law. However, it was first derived from general relativity by Georges Lemaître in 1927. Hubble did observations on the subject that confirmed Lemaître.

Hubble's Law describes the Hubble motion which is caused by the expansion of the Universe. This law states that the relative speed of a body caused by the expansion is a function of the **Hubble constant** (H_0) and the distance. The Hubble constant is the speed of variation of the scale factor of the Universe (the dimensionless factor that increases distances on a certain scale).

However, we know that the Hubble constant (H_0) is time dependent and therefore, it is called Hubble parameter (H) when we assume any other time that is not the time of observation. The term Hubble constant (H_0) is only used when describing the Hubble parameter at $t=0$, meaning at the time of observation or now. The exact value of the Hubble constant is unknown, but it is thought to be around 70 (km/s)/Mpc*. Which means that for each Mpc that a body is further away from us, its recession velocity increases by 70 km/s. Now follows the mathematical justification of this principle.

We know that the variation over time of the distance or separation of two points is described by:

$$d(t) = a(t)d_0 \tag{3}$$

Where d_0 is the initial distance, $a(t)$ is the dimensionless scale factor of the expansion and $d(t)$ is the new distance.

If we now derivate this expression with respect to time we obtain:

$$\frac{dd(t)}{dt} = \frac{da(t)}{dt}d_o$$

$$v(t) = \dot{a}(t)d_0 \tag{4}$$

Where $v(t)$ is the expansion speed, $\dot{a}(t)$ represents the derivative of the scale factor and d_0 the initial distance. By convention, in cosmology the dot on top of a variable represents the derivative for time of this variable. $\dot{a}(t)$ is not a dimensionless quantity, it has units.

One could argue that at far enough distances the recession speed could get faster than the speed of light and so violate special relativity. Although this can happen, it would not violate special relativity, because the object would not be moving faster than the speed of light, space itself would be moving.

We know that the Hubble parameter is described by the following expression:

$$H = \frac{\dot{a}(t)}{a(t)} \tag{5}$$

*A megaparsec is a unit for space used to measure big distances outside of the Solar System.
1 Mpc ≈ 3.1 x 10^{22} m

And if we set *t=0* the following happens:

$$d(0) = a(0)d_0$$

$$a(0) = 1$$

$$H = \frac{\dot{a}(0)}{a(0)} = \frac{\dot{a}(0)}{1} = \dot{a}(0)$$

$$H_0 = \dot{a}(0) \tag{6}$$

We apply this to *Equation 4* and:

$$v(0) = H_0 d_0 \tag{7}$$

Now we can see that the Hubble constant equals to the derivative of the scale factor at *t=0*, meaning the time of observation. One application of this could be that if we now the exact Hubble parameter and the recession velocity of a body (which can be determined relatively easily), we can determine its exact distance from us and vice versa.

However, the most detailed description of the expansion of our universe is given by the **Friedmann equations** derived by Alexander Friedmann. Let's derive its standard form:

First, knowing that mass is the product of volume and density as well as that the volume of a spherical object is described by $V=(4\pi d^3)/3$, we can derive the following expression:

$$M = \frac{4\pi d^3 \rho}{3} \tag{8}$$

Then, assuming Newtonian gravity, we substitute M into *Equation 2*:

$$E_{Pg} = -\frac{GMm}{d} = -\frac{Gm4\pi d^2 \rho}{3} = -\frac{Gm4\pi a^2 {d_0}^2 \rho}{3} \tag{9}$$

Now we apply the formula for kinetic energy to determine the kinetic energy of the Hubble motion using *Equation 4*:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{a}^2 {d_0}^2 \tag{10}$$

Now we can apply the law of conservation of energy:

$$U = E_K + E_{Pg} = \frac{1}{2} m\dot{a}^2 {d_0}^2 - \frac{Gm4\pi a^2 {d_0}^2 \rho}{3} \tag{11}$$

Which, after some algebra, looks like this:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G {d_0}^2}{3} \rho - \frac{kc^2}{a^2} \tag{12}$$

Where H is the Hubble parameter, G is the gravitational constant, a is the scale factor of the expansion of the Universe, c is the speed of light, d_0 is the initial distance and $kc^2 = -2U/(m \cdot d_0^2)$.

Equation 12 is the standard form of the Friedmann equations. The Friedmann equations can appear in many ways, some of them are going to be used during this project, but they are not going to be derived.

The value k is a very important value. It is a constant that is unique to every expanding Universe and plays an important role in the geometry of our universe, determining its curvature.

The ρ term of the Friedmann equation represents the density of the material in our universe. It is time dependent, meaning that it changes over time. It is described by the **fluid equation**:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0 \tag{13}$$

Where ρ is density (mass/volume), p is pressure (force/area), c is the speed of light and a is the scale factor of the expansion of the Universe. The ρ in brackets accounts for the density dilution as the volume increases. The p/c^2 accounts for the energy loss due to the work the pressure has done as the Universe's volume increased.

For clarification, this pressure does not cause a force and is not making the Universe expand, since density and pressure is the same everywhere.

Another important relationship between the pressure and the density is the **equation of state** (w_0). The equation of state (w_0) is characterized by a dimensionless value:

$$w_0 = \frac{p}{\rho} \tag{14}$$

Where w_0 is a dimensionless ratio, p is pressure and ρ is density. The equation of state has the following effect on the scale factor (see *Graph 2*, page 30):

$$\rho \propto a^{-3(w_0+1)} \tag{15}$$

We can apply this equation to matter and radiation:

- Matter or non-relativistic matter doesn't exert any pressure. Therefore, $p=0$ or $w_0=0$, so $\rho \propto a^{-3}$.
- Radiation exerts a pressure described by $p=\rho/3$ or $w_0=1/3$, so $\rho \propto a^{-4}$.

2.3.3. Geometries of the Universe

Always having in mind that our universe have to be homogenous and isotropic, there are three possible types and geometries our universe could have, depending on the value of *k*, as shown in *Table 1* and *Image 12*. However, this only talks about the geometry of three-dimensional space. If we wanted to know the geometry of four-dimensional space-time we would need to apply general relativity, which we won't do in this project.

Curvature	Geometry	Type of universe
$k>0$	Spherical	Closed
$k=0$	Flat	Flat
$k<0$	Hyperbolic	Open

Table 1. Table of the possible geometries and types of universe depending on the value of k. Source: *An Introduction to Modern Cosmology* by A. Liddle.

It has been tried to prove what kind of curvature our universe has and in most cases it was found that the value of k has to be close to 0. However, the errors don't allow us to take it as a solid prove, since the slightest deviation from 0 would mean that the Universe isn't flat.

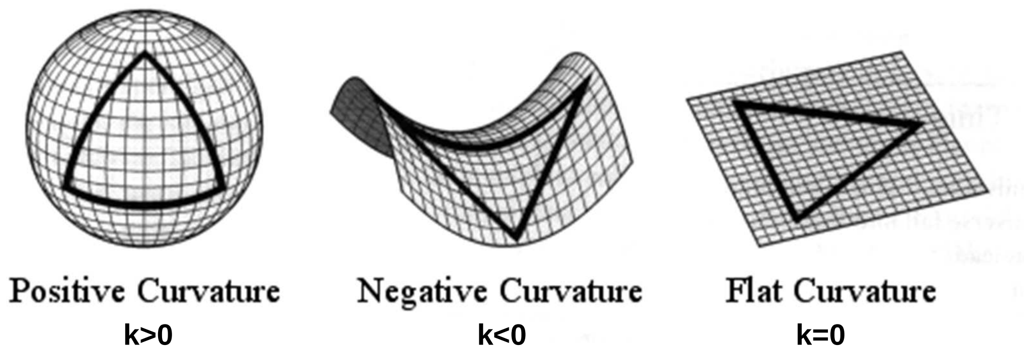


Image 12. Graphic representation of the three possible types of space curvature depending on the value of k. Source: *abyss.uoregon.edu*

Whether the Universe is infinite or not depends on how you define infinite. Usually we mean by infinite that it is infinite in space, but finite in time. We usually talk more about the **observable universe**. This is the part of the Universe from which light had time to reach us. The observable universe is probably never going to include the entire Universe, since it is very big or even infinite and is expanding (at some distances even faster than light).

2.3.4. Cosmological Models

a. Cosmological Constant

When Albert Einstein formulated general relativity, the most accepted cosmological theory was for a static universe. This means that the Universe was still, it wasn't expanding or getting smaller. Nevertheless, general relativity didn't allow a static universe, because the pull of gravity would make it collapse. Thus, Einstein introduced the **cosmological constant** (Λ), a force that would counteract against gravity in order to keep the Universe static. Later Einstein called the cosmological constant his "greatest blunder".

Today we know that the Universe is expanding and we know that it is accelerating (*which we will prove later*). Therefore, the static universe theory is debunked, but the cosmological constant is still in use. So far, this constant is one of the most appealing explanations we have for the expansion and acceleration of space. Now we think it doesn't only counteract gravity, it overcomes it and so makes the Universe expand and accelerate. We don't know what the cosmological constant is, not even if it is a constant or not, but it could be dark energy (see 2.4.3.).

If we now introduce this constant into the Friedmann equation from *Equation 12,* it looks something like this:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \tag{16}$$

Where *H* is the Hubble parameter, *G* is the gravitational constant, *ρ* is density, *k* is the curvature constant, *c* is the speed of light, *a* is the scale factor and *Λ* is the cosmological constant.

If we now add this term to the acceleration equation (the derivative of the Friedmann equation that describes the acceleration of the Universe):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \tag{17}$$

Where *a* is the scale factor, *G* is the gravitational constant, *ρ* is density, *p* is pressure, *c* is the speed of light and *Λ* is the cosmological constant.

This equation clearly shows that the cosmological constant can counteract the pull of gravity. If it didn't exist, were negative or very small the Universe would decelerate and eventually collapse. If it were positive and equal to the gravitational attraction, it would cause a static universe. However, this universe would be very unstable and is not possible in practice. Finally, if the cosmological constant is positive and bigger than the gravitational attraction, it could overcome it and cause the expansion of the Universe to accelerate.

The cosmological models that use the cosmological constant (*Λ*) and dark matter (see 2.4.2.) to describe the Universe are called **ΛCDM models** (Lambda cold dark matter models). They are the simplest models to account for the the large-smoothness (see 2.3.1.a.), the Big Bang (see 2.3.4.b.) and the acceleration of the Universe. These are also the models that are going to be used in the practical part of this project.

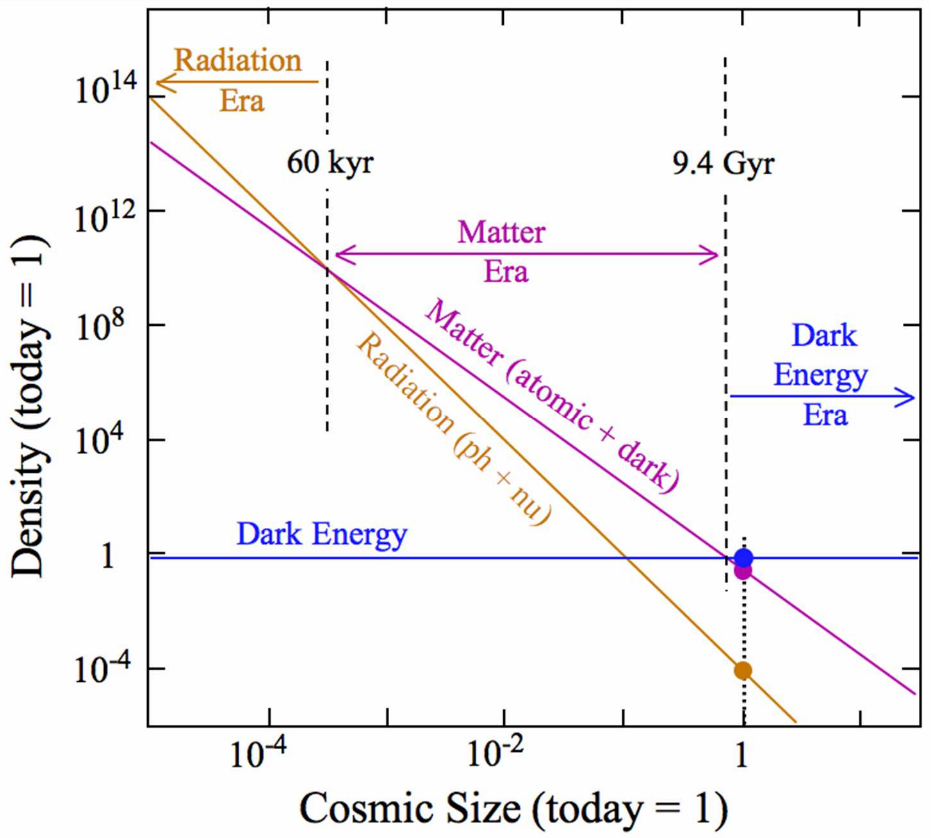
b. The Big Bang Theory & the Inflationary Model

The **Big Bang Theory** is the most widely accepted cosmological model that describes the origins of our universe. It says that at some point when time didn't exist, all space was compressed in one infinitely small point called a singularity. This singularity eventually exploded in the Big Bang and started the expansion of space and was the beginning of time. After the Big Bang, the Universe underwent several processes:

- **Inflation** (approx. 10^{-36} s - 10^{-32} s): According to the inflationary model, right after the Big Bang, the Universe expanded exponentially for a very short period of time. At this time the Universe consisted of a soup of elementary particles where no structures had formed yet.
- **Cooling** (approx. 10^{-32} s – 300,000 years): After inflation stopped, the Universe continued expanding and started cooling. The Big Bang probably created an almost equal amounts of matter and anti-matter which annulated and formed a lot of high energy electromagnetic radiation. Until 60,000 years after the Big Bang, the Universe was radiation dominated and the expansion was governed by radiation. The expansion rate of the Universe while being governed by radiation was relatively slow. Then, when the matter density of the Universe exceeded the radiation density, matter started governing the expansion of the Universe. The matter dominated Universe expanded a bit quicker due to a slower deceleration, but asymptotically. The Universe had still been matter dominated until recently, but the cooling phase of the Universe finished about 300,000 years after the Big Bang.
- **Structure forming** (approx. 300,000 years – today): This phase starts when the matter present in the Universe gets cool enough to form stable atoms. Until then, because of the high temperatures, all atoms were ionized and nuclei and electrons existed independently of each other. This didn't allow light to get through the Universe, since the photons interacted with the electrons. The structure forming phase starts with the appearance of the first light that could travel far distances and eventually reach us.

The light that reaches us from the start of that period is called the Cosmic Microwave Background (see 2.4.1). Although this radiation started as high energy gamma rays, only low energy microwaves reach us, because the expansion of the Universe stretched their wavelength.

Since matter wasn't contained in a plasma soup anymore, it could start clumping together. During this period is when the first galaxies appeared. We could say that structure forming is still occurring today. However, we would have to make one distinction: until "recently" the Universe was matter dominated and the expansion of the Universe was governed and decelerated by matter, but, as the matter density continued decreasing, at some point the dark energy (see 2.4.3.) density took over and started governing the expansion of the Universe.

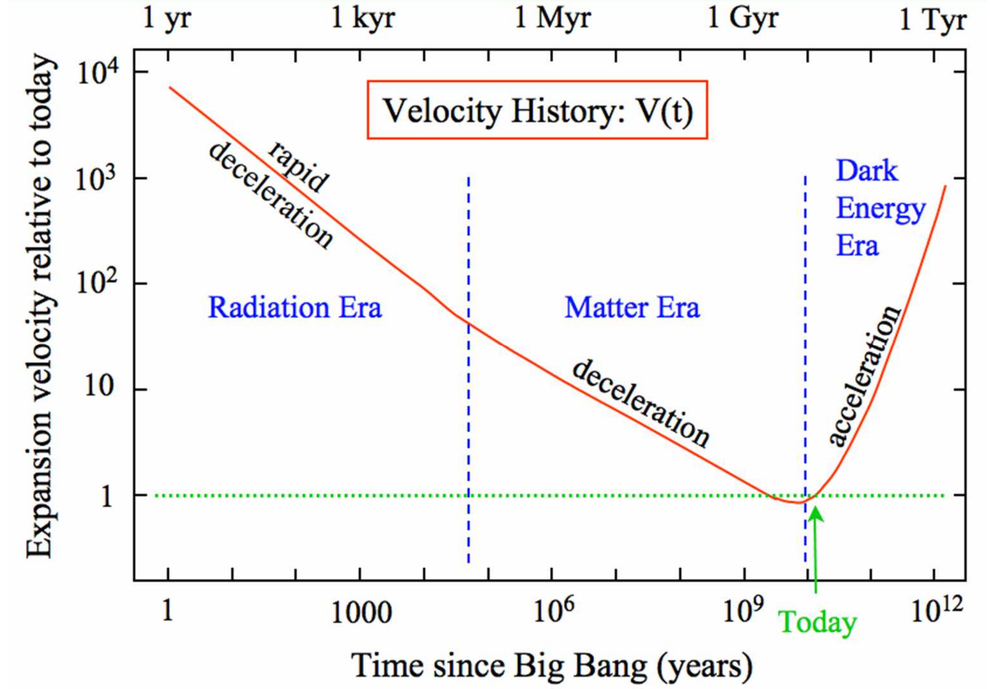


Graph 1. Evolution of the density of the three basic cosmic components: radiation (photons & neutrinos), matter (dark & baryonic), and dark energy (vacuum). The colored dots show their current densities. Whichever dominates defines the era. Source: *www.astro.virginia.edu*

- **Cosmic expansion**: When the matter era finished, the Universe was getting dominated by the cosmological constant or dark energy (if it is truly a constant). This meant that the expansion of the Universe didn't progress slowly anymore and started increasing exponentially. Currently we are experiencing the cosmic expansion, since most observational evidence points towards an accelerating universe.

  Cosmic expansion is eventually going to lead to such a fast expansion speed that every galaxy is going to move away from all the others faster than the speed of light. This means that the light emitted from those galaxies and stars will never reach us or any other point in the Universe. Then, if the expansion speed gets even faster we might get to a point where gravity won't even be able to hold the closest bodies together and structure forming will stop.

  Graph 1 classifies the time after the Big Bang depending by what material it was dominated and *Graph 2* describes the expansion of the Universe during each of those eras.



Graph 2. Evolution of the expansion velocity relative to today, depending on what cosmic component is dominating the Universe. Source: www.astro.virginia.edu

2.3.5. Observations

a. Observational Parameters

Modern cosmology requires a lot of experimental work. Many values we have seen so far can only be determined with astronomical observations.

One value we can measure almost directly is the **redshift** (*z*). Redshift can be defined as the stretching of the wavelength of a light beam, due to the expansion of the Universe or because the light source is moving away from the observer. It is called redshift because the wavelength increases, thereby shifting the light towards the red end of the spectrum.

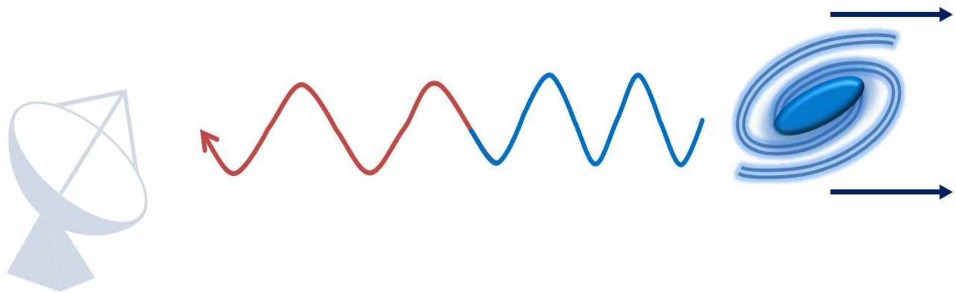


Image 13. Diagram of the gradual redshift caused by the expansion of the Universe. Source: *www.skatelescope.org*

Redshift can be described by the following formula:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \tag{18}$$

Where z is redshift, λ_{obs} is the observed wavelength and λ_{em} is the original wavelength emitted by the body.

Redshift can also be described as the ratio between the recession speed (v) and the speed of light (c):

$$z = \frac{v}{c} \tag{19}$$

However, this only holds true if $v \ll c$. If the bodies are further away and the recession speed is greater, we will have to take special relativity into consideration:

$$1 + z = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \qquad (20)$$

This relationship between redshift and the recession speed of the body brings us one step closer to figuring out the Hubble constant, since it is related to the recession speed by Hubble's law (*Equation 7*).

Another observational value we can obtain is **luminosity distance** (D_L). Luminosity distance is nothing else but the distance of the object in megaparsecs determined from its absolute and apparent magnitude. The absolute magnitude is the relative brightness a body really has and the apparent magnitude is the relative brightness the light from the body has when it gets to us. Luminosity distance can be obtained from the following logarithmic function:

$$D_L = 10^{\frac{(m-M)}{5}+1} \qquad (21)$$

Where D_L is distance luminosity, m is the apparent magnitude and M is the absolute magnitude.

Now we have everything we need to calculate the Hubble parameter. Nevertheless, one problem arises. Both, redshift and luminosity distance, depend on an initial value: emitted wavelength and absolute magnitude, respectively. Finding a way of determining those initial values was a problem that intrigued scientists for a long time. Eventually, a solution was found. It was found that there are certain types of supernovae – the explosions that occur when a big star dies – that at a certain point always emit light at the same magnitude and wavelength. These supernovae are called type 1a supernovae and always occur in binary star systems. Thanks to those standard candles – the term used for bodies that always present the same magnitude no matter where they are in space – cosmologists can now generalize their emitted wavelength as well as absolute magnitude and thereby calculate the Hubble constant.

A very important observational parameter that will appear in this project is the **density parameter** (Ω). It can be seen as a help to picture the density term (ρ) shown in chapter 2.3.2. It can be described by the following formula:

$$\Omega = \frac{\rho}{\rho_c} \tag{22}$$

Where Ω is the density parameter, ρ is the current density and ρ_c is the critical density. All variables are time dependent, but the present value of Ω is sometimes denoted Ω_0.

The critical density (ρ_c) is the density needed for making the $k=0$ in the Friedmann equation (*Equation 12*), so that the Universe has a flat geometry. Nevertheless, the use of this term does not imply that the Universe is flat. It is just a reference point in order to make the density parameter physically informative, as shown by *Table 2*.

Density	Density parameter	Curvature	Type of universe	Geometry of universe
$\rho>\rho_c$	$\Omega>1$	$k>0$	Closed	Spherical
$\rho=\rho_c$	$\Omega=1$	$k=0$	Flat	Flat
$\rho<\rho_c$	$\Omega<1$	$k<0$	Open	Hyperbolic

Table 2. Table of the possible types of Universe that shows the relationship between density, the density parameter and curvature. Source: Own

The value of the critical density today ($\rho_c(0)$) is approximately equal to 9.27 x 10^{-27} kg/m^3. This might not sound like much, but it is approximately the density we have today, since the current value of Ω_0 is measured to be at (1.02±0.02). This is very close to a flat universe. Therefore, in the practical part a flat universe will be assumed.

The density parameter can also be decomposed into the different constituents of the Universe: matter, radiation and dark energy. Now we can say that:

$$\Omega = \Omega_M + \Omega_\Lambda + \Omega_R \tag{23}$$

Where Ω is the total density parameter, Ω_M is the matter density parameter (including dark matter, see 2.4.2.), Ω_Λ is the dark energy density parameter and Ω_R is the radiation density parameter. However, in most cases, the radiation density parameter is seen as negligible, since $\Omega_R \approx 0$ today:

$$\Omega_0 = \Omega_M + \Omega_\Lambda \tag{24}$$

Lastly, from all this we can see that the density parameter can be determined experimentally, because it is proportional to density which we can relate to the Hubble parameter with the Friedmann equation.

b. Instruments

All those models could never be proven if we wouldn't have instruments to conduct experiments or observations. These instruments are usually telescopes. But telescopes aren't only the relatively small tubes with lenses most of us already used. They can be of many types and can operate on many different wavelengths of the electromagnetic spectrum (*Image 8*).

One very important thing for telescopes is their location. Depending on where they are located they can get better observations. The most important factors to take into account are air disturbance and light pollution.

First, since our planet has an atmosphere, the light that reaches our telescopes gets refracted by the air and gets slightly diffused. This can cause imprecise measurements. Therefore, most high precision telescopes are located in high altitudes where there is less air and the air flow is more linear. The best way to overcome this problem is to locate telescopes in an environment without air, space. Sending telescopes into space allows higher precision in the measurements, but facing very high costs.

Image 14. The Atacama Large Millimeter/submillimeter Array operating at night. Source: *www.soychile.cl*

Another big problem astronomers face when they have to pick the location for their telescope is light pollution. Light pollution is the rest light in our atmosphere emitted by humans. This is why telescopes are usually located in desserts or on islands.

Some of the most famous telescopes in use right now are the Hubble Space Telescope, the Planck Space Telescope, ALMA (Atacama Large Millimeter/ submillimeter Array), the VLT (Very Large Telescope) or the GTC (*Gran Telescopio Canaria*s).

2.4. Recent discoveries

2.4.1. Cosmic Microwave Background

The **Cosmic Microwave Background**, as mentioned in chapter 2.3.4.b., is the first light created after the Big Bang that could arrive to us. It appeared after the Universe had cooled down enough to allow the formation of atoms and to allow light to pass through space without interacting with electrons on the way.

Even though this phenomenon had been theorized in the 1940s, it was not until 1964 when the first clear measurement of the CMB was made by Arno Penzias and Robert Woodrow Wilson. For the observation and prove of the CMB, they received the 1978 Nobel Prize in Physics.

The average “temperature” of the CMB is (2.72548±0.00057) K and seems to be very regular, but it still shows some minimal variations. Those small variations might be the causes of some large scale differences in the Universe today and are very interesting for cosmologists.

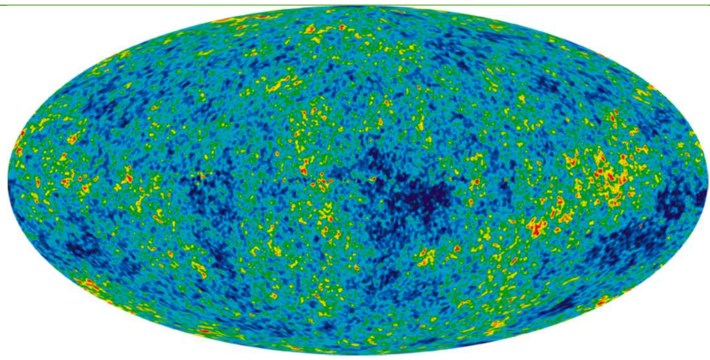


Image 15. Representation of the CMB data obtained by the WMAP survey showing the temperature variations (blue means cold and red means warm). Source: *en.wikipedia.org*

2.4.2. Dark Matter

Dark matter is a very abundant component of our universe that possesses mass and doesn't interact with regular matter or radiation. It has not been seen or detected directly yet, but its gravitational effect has been observed. Therefore, we don't know what it is made of.

The idea of dark matter appeared when scientists observed a discrepancy between the expected rotation speed of galaxies and the real one. The rotation speed galaxies should have was derived from the mass of the stars and other bodies that the galaxies contained. However, in order for the galaxy to achieve the measured rotation speed and at the same time not fall apart, it needed to have more mass than visible. This is when the idea of dark matter appeared during the 1960s and 1970s.

Until today there has been way more evidence for the existence of dark matter found in the CMB or the movement of galaxy clusters. Many theoretical work is being done to try to describe what dark matter is and experiments are being designed to observe it directly.

2.4.3. Dark Energy

Dark energy is the vacuum force that fills all space and counteracts the pull of gravity in order to accelerate the expansion rate of the Universe. Currently it is mostly accepted that it is a field of constant density everywhere in the Universe that doesn't interact with regular matter or radiation. Its exact nature is still unknown, since it could still be some other kind of field and not have a constant density.

The idea of dark energy appeared as an explanation for the acceleration of the expansion of the Universe discovered by the High-Z Supernova Search Team and the Supernova Cosmology Project in 1998. For this discovery members of both research groups received the 2011 Nobel Prize in Physics.

A more detailed prove of the nature of dark energy will be provided in the practical part of this project.

3. PRACTICAL PART

3.1. Introduction

Until now we have seen what modern cosmology is, its origins and some of the topics that are studied with it. However, how does a modern day cosmologist work? To answer this question we will work on a current research topic and we will use the methods a cosmologist would use to study it.

In the practical part of this project we will see how starting with some data sets of some measured parameters, we can calculate other values using the ΛCDM models (see 2.3.4.a.) and from there derive "new" physics. Even though all of what is being shown in this project has already been done and discovered by scientists, we will do it from scratch as if we were the scientists trying to discover new physics.

The mentioned new physics, as foreshadowed earlier, is the acceleration of the expansion of the Universe and dark energy. Using Type 1a Supernova data from the *Supernova Cosmology Project* compilation Union 2.1 (see 8.1. Annex A) we will constrain the matter content of our universe and try to prove the existence of a vacuum force or dark energy.

Since the data set is a matrix (data table) of many values, the calculations can't be done by hand. Therefore, we will use MatLab. Matrix Laboratory is a program that allows us to write codes in order to manipulate data with a certain algorithm. Only the steps of the algorithm and the graphs or tables generated from it, are going to appear in this practical part. The code itself and a flux diagram can be seen in Annex B (see 8.2.).

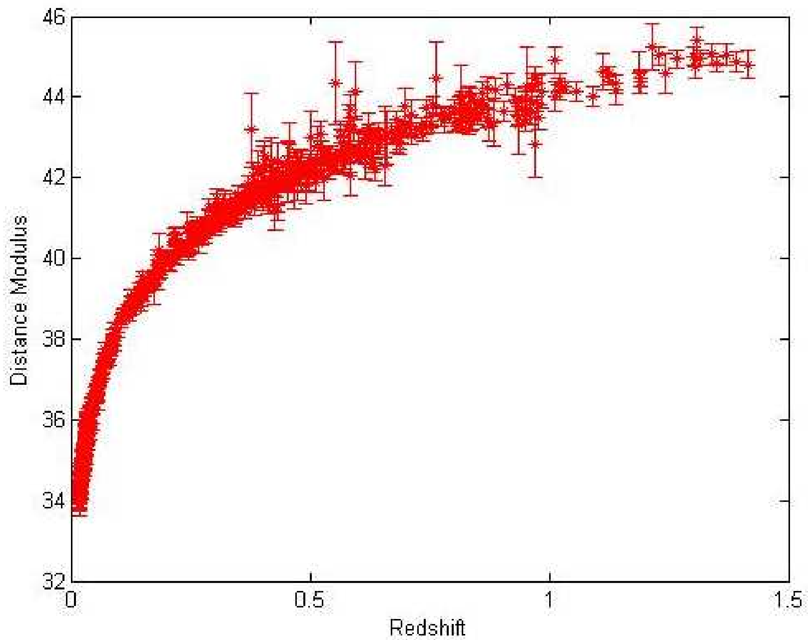
When we operate with those measured values, we will also have to operate their errors and likelihoods. To do that we will use some Bayesian statistics. The formulas might seem scary, but they only try to compensate the inaccuracies in the data by generating a probability of some proposition being correct. This means that something that could only be right or wrong, now, with Bayesian statistics, it can have a certain probability of being correct. More information about statistics can be found in the book *Probability and Statistics. The Science of Uncertainty* by Michael J. Evans and Jeffrey S. Rosenthal

3.2. The Expansion and Acceleration of the Universe

First, let's have look at our data set. The data (see 8.1. Annex A) has been obtained by the Supernova Cosmology Project from the Berkeley Lab (USA) using wide-field telescopes to find the supernovae and a few other telescopes to analyse the spectra and to do the photometry of the supernovae. It can be found on their website: *supernova.lbl.gov*. The dataset gives us the name of the supernovae, their redshift (z) (see 2.3.5.a.) and their distance modulus (μ) with its error ($\Delta\mu$). Distance modulus can be defined as:

$$\mu = m - M \tag{25}$$

Where μ is the distance modulus, m is the apparent magnitude and M is the absolute magnitude. All 580 data points can be seen in *Graph 3*.



Graph 3. Graph of distance modulus (μ) with errors vs. redshift (z) obtained from the Supernova Cosmology Project data set. Source: Own

The simple fact that as bigger the distance modulus (μ) gets its light is more redshifted proves the **expansion** of the Universe. Because if the Universe wasn't expanding there would hardly be any redshift (z) and it would most certainly not be proportional to the distance. However, this plot doesn't tell us much more for now. It's just a visualization of the data we have. In order to obtain more information from this data we have to convert de distance modulus

(μ) and its error ($\Delta\mu$) into luminosity distance (D_L) and its error (ΔD_L). To get the luminosity distance we can use *Equation 21* to get:

$$D_L = 10^{\frac{\mu-25}{5}} \tag{26}$$

To obtain the luminosity distance error (ΔD_L) from the distance modulus error ($\Delta\mu$), we use the error propagation formula for the function $f(x) = Ae^{\pm Bx}$:

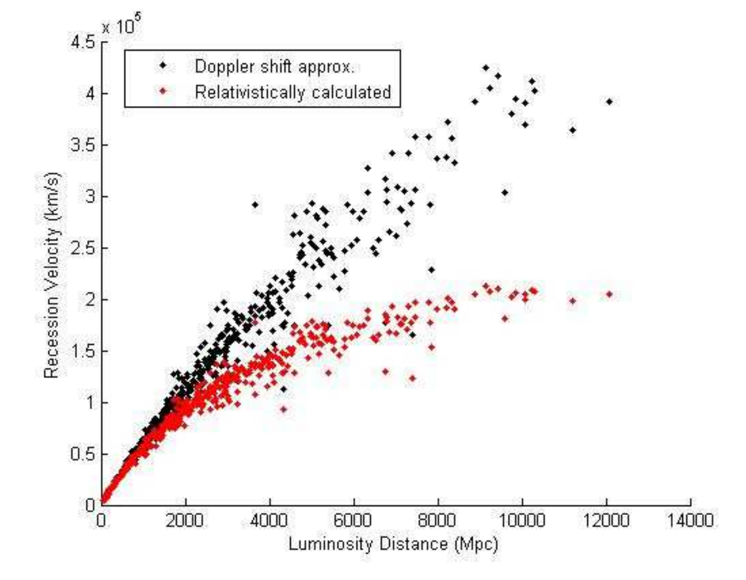
$$\sigma_f = B\sigma_x f(x) \tag{27}$$

Where σ_f is the error of the function, *B* a constant factor, σ_x is the error of the variable and *f(x)* is the function. For our function of luminosity distance it should look something like this:

$$\Delta D_L = \frac{\Delta\mu\, D_L}{5} \tag{28}$$

Where ΔD_L is the error of luminosity distance, $\Delta\mu$ is the error of distance modulus and D_L is the luminosity distance.

We also have to convert redshift (*z*) into recession speed (*v*). As we saw in 2.3.5.a., there is a clear relationship between them that can be described with a Doppler shift approximation (*Equation 19*) or using special relativity (*Equation 20*). We know that for this phenomenon using special relativity would be more correct and in *Graph 4* we can see that when $v>0.5 \times 10^5$ *km/s* there is a noticeable discrepancy between the two descriptions. Thus, the values used from now on come from this relativistic formula that was derived from *Equation 20*:

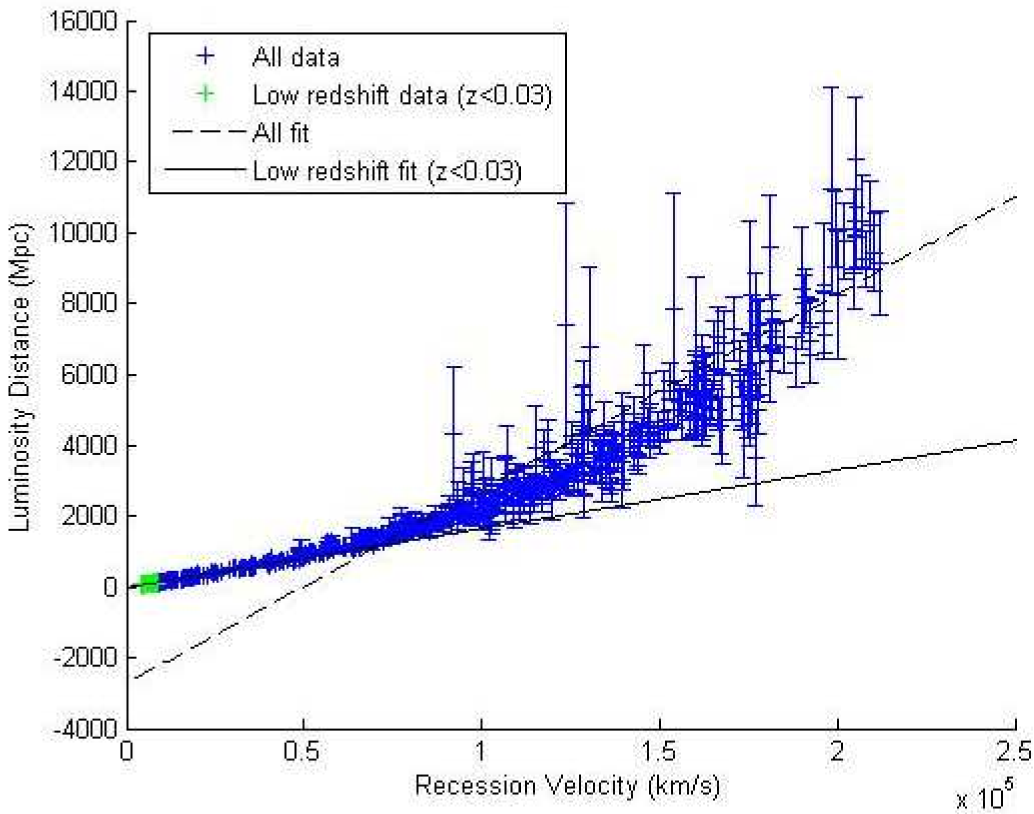


Graph 4. Graph of recession velocity (km/s) vs. luminosity distance (Mpc) comparing relativistic data to approximated data. Source: Own

$$v = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1} \tag{29}$$

Where *v* is the recession speed, *c* is the speed of light and *z* is the redshift.

If we now plot the data again and make a best fit line for all the data and for the data obtained from values of *z<0.03*, we obtain *Graph 5*.



Graph 5. Plot of luminosity distance (Mpc) vs. recession velocity (km/s) with linear best fit lines of all the data and the most recent data (*z<0.03*). Source: Own

In *Graph 5* appears a linear best fit line for all the data (the discontinuous straight line that best describes the behaviour of all the data points). However, we can see that his line doesn't perfectly follow the path of the data points. Thus, the approximation is incorrect and the appropriate best fit line should be a curve. We know that all curves show a variation in their slope. This means that in this case the Hubble parameter (*H*), which according to Hubble's law is the inverse of the slope of this graph, changes with the recession speed (*v*) which is a function of redshift (*z*) which varies with time. This means that *H* (the expansion "speed" of the Universe) increases with time and, therefore, the expansion of the Universe is **accelerating**.

Another relevant aspect of *Graph 5* is that the slope of the best fit line of all data with a redshift below 0.03 is approximately equal to the Hubble constant. As we discussed in 2.3.2., the Hubble constant is the Hubble parameter at *t=0*. Of course, if we take values of z that are bigger than zero, they can never be at *t=0*, but they are a good approximation, since the variation in that interval isn't too big. The function for the best fit line of the low redshift (the continuous line) data is:

$$f(x) = mx + n \tag{30}$$

It falls within the following interval:

$$f(x) = \ (0.01492, 0.01832)x + (-20.94, 1.784) \tag{31}$$

We now can calculate the Hubble constant (H_0) using the median of the slope interval ($\overline{m}$):

$$H_0 = \frac{1}{\overline{m}} = \frac{1}{0.01662} = 60.168\ km\ s^{-1}Mpc^{-1} \tag{32}$$

For the errors we apply:

$$\Delta H_0 = \frac{m_{max} - m_{min}}{2\overline{m}^2} = \frac{0.01832 - 0.01492}{2(0.01662)^2} = 6.154\ km\ s^{-1}Mpc^{-1} \tag{33}$$

Where ΔH_0 is the error of the Hubble constant, m_{max} is the maximum slope of the best fit line, m_{min} is the minimum slope of the best fit line and $\overline{m}$ is the median of the interval of the slope.

We have obtained an approximation of the Hubble constant:

$$H_0 = (60.168\ \pm 6.154)\ km\ s^{-1}Mpc^{-1} \tag{34}$$

3.3. Dark Energy

So far we came to the conclusion that the rate of expansion of the Universe is not constant, so there should be some kind of acceleration. In addition, this acceleration is positive, so there must be a force that is overcoming the pull of gravity. So let's have a look at the acceleration equation (*Equation 17*) derived from the standard Friedmann equation (*Equation 16*):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \qquad (35)$$

There we can see that the only way for the acceleration to be positive, $\frac{\ddot{a}}{a} > 0$, is when the gravity term ($-\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$) is smaller than the cosmological constant term ($\frac{\Lambda}{3}$):

$$\left|-\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)\right| < \left|\frac{\Lambda}{3}\right| \qquad (36)$$

However, we don't know what behind this cosmological constant is nor if it really is a constant or if it is maybe a function of time. To figure this out we need another version of the Friedmann equation that uses the density parameters:

$$H(z) = H_0\sqrt{\Omega_M(1 + z)^3 + \Omega_R(1 + z)^4 + \Omega_\Lambda} \qquad (37)$$

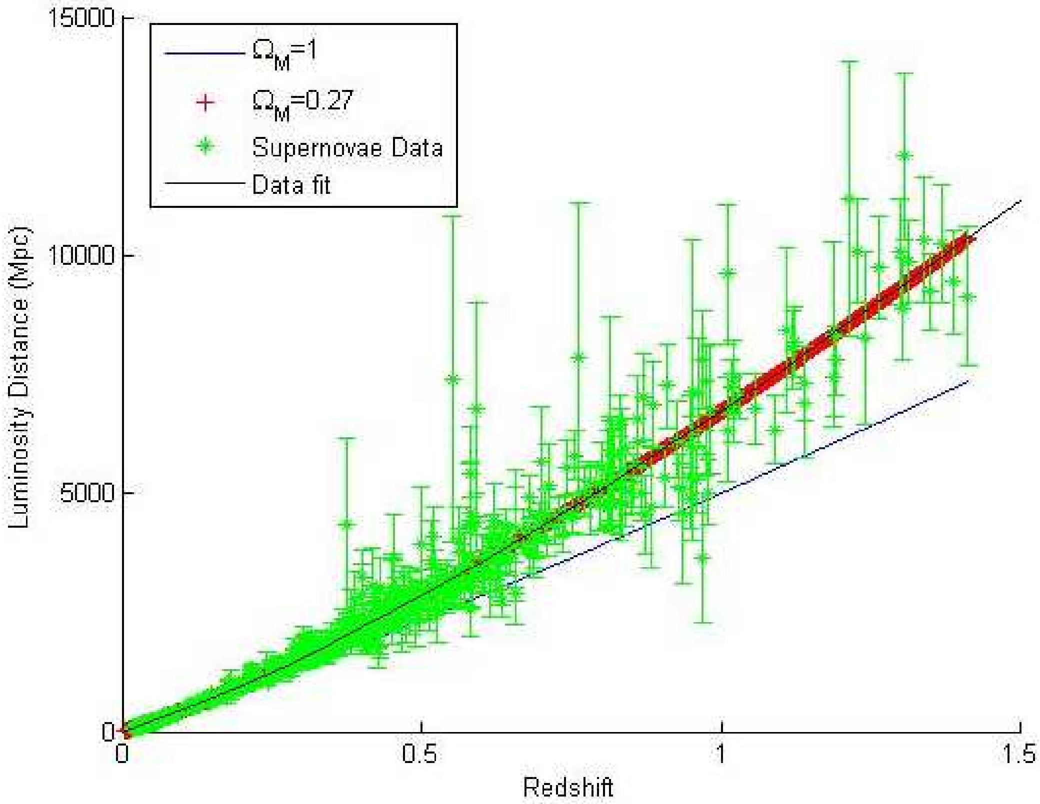
Where *H(z)* is the Hubble parameter as a function of redshift, H_0 is the Hubble constant, Ω_M is the matter density parameter, Ω_R is the radiation density parameter and Ω_Λ is the cosmological constant density parameter. This formula tells us that only the influence of matter and radiation on the Hubble parameter (*H(z)*) varies with redshift (*z*), since their density isn't constant and changes with time, just like redshift (*z*). The cosmological constant density parameter (Ω_Λ) doesn't have a factor that depends on redshift (*z*), because we are assuming that it has a constant density everywhere in the Universe.

If we now use *Equation 37* with Hubble's law and make $\Omega_R \approx 0$, we obtain luminosity distance (D_L) as a function of redshift (*z*), the Hubble constant (H_0), the matter density parameter (Ω_M) and the cosmological constant density parameter (Ω_Λ):

$$D_L = c(1+z)\int_0^z \frac{dz}{H(z)} = \frac{c}{H_0}(1+z)\int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}} \qquad (38)$$

By applying this equation to the data and assuming a flat universe ($\Omega_M + \Omega_\Lambda = 1$), we obtained *Graph 6*. The graph shows a plot of our luminosity distance (D_L) data vs. the redshift (*z*). Also using *Equation 38*, we computed the predicted best fit line for the data if $\Omega_M=1$ and $\Omega_\Lambda=0$ (no cosmological constant, blue line) and we computed another best fit line for $\Omega_M=0.27$ and $\Omega_\Lambda=0.73$ (red line), which is thought value for the parameters according to other studies. Then, we can see that the best fit line for the data (black line) coincides with the theoretical line of Ω_M=0.27 and Ω_Λ=0.73 (red line). This means that the real value of Ω_M and Ω_Λ must be somewhere around this value.

In conclusion, we can say that, assuming H_0=70 km s^{-1} Mpc^{-1} and a flat universe ($\Omega_M + \Omega_\Lambda = 1$), around 73% of the energy content of the Universe should be made of cosmological constant or dark energy. Therefore, we refuted the model of a flat and matter-dominated universe (Ω_M=1) that was the most accepted until 1998, because the data is clearly deviating from the Ω_M=1 line.



Graph 6. Plot of luminosity distance (D_L) in megaparsecs vs. redshift (z) assuming a flat universe and H_0=70 km s^{-1} Mpc^{-1}. There is a theoretical line for Ω_M=1 in blue and another one for Ω_M=0.27 in red. The black best fit line for the data coincides with the red line for Ω_M=0.27. Source: Own

Nevertheless, we can't make a too exact claim about the value of the density parameters, since with our data and its errors there is a whole possible range of values. In addition, we don't know the exact value of the Hubble constant (H_0), which could also make the values vary. Thus, we will have to do a likelihood analysis of the likelihood each possible value of Ω_M and Ω_Λ has to be the correct one.

In order to that we use the Bayesian likelihood function for 580 data points:

$$\mathcal{L}(j) = exp\left\{-\frac{1}{2}\sum_{i=1}^{580}\frac{[D_{Ldat}(i) - D_{Lth}(i,j)]^2}{\Delta D_L(i)^2}\right\} \quad (39)$$

Where $\mathcal{L}(j)$ is the likelihood as a function of *j* (the resolution), *exp* is equal to e^x, D_{Ldat} is the luminosity distance from the data, D_{Lth} is the luminosity distance obtained theoretically and ΔD_L is the error of the luminosity distance data.

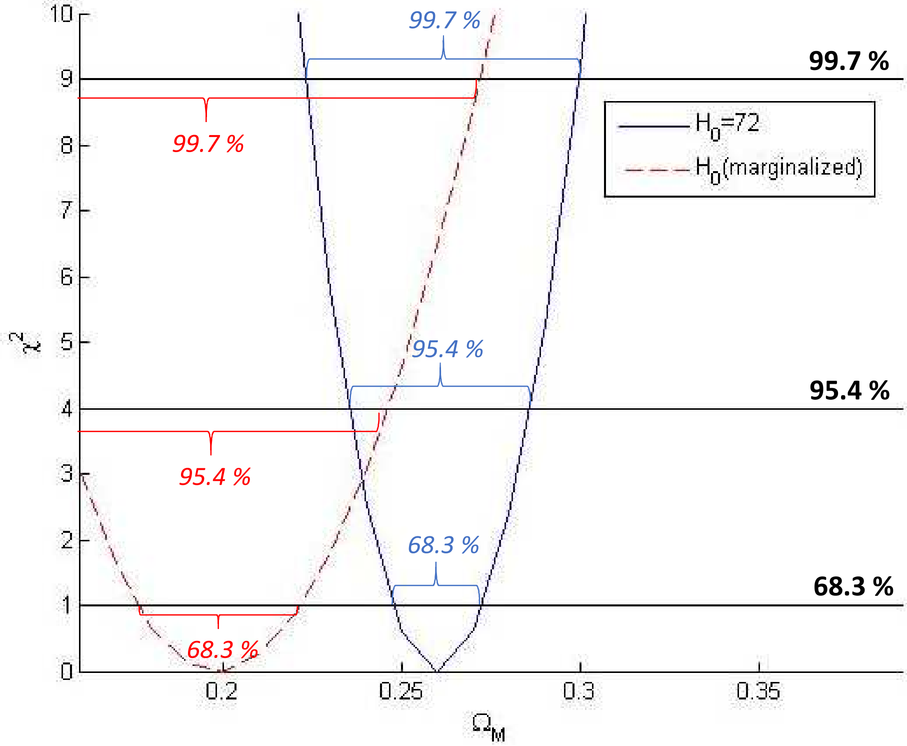
However, the likelihood value is a little too hard to visualize. Therefore, we will use the chi squared value (X^2) in order to have a smaller and simpler number:

$$\chi^2 = -2\ln\mathcal{L}(j) = \sum_{i=1}^{580}\frac{[D_{Ldat}(i) - D_{Lth}(i,j)]^2}{\Delta D_L(i)^2} \quad (40)$$

Since the luminosity distance is still a function of the Hubble constant (H_0) we need to set a prior* for it. We used two different priors for *Graph 7*. First, we used a fixed prior at $H_0=72\ km\ s^{-1}\ Mpc^{-1}$. Second, we also did the likelihood calculation using a marginalized prior for H_0. This means that several values of the Hubble constant inside a set interval are all used for one likelihood. The set range for this marginalization was $H_0= (70, 74)\ km\ s^{-1}\ Mpc^{-1}$ where each value was weighted equally and only natural values were used.

Graph 7 tells us that using our data and setting $H_0=72\ km\ s^{-1}\ Mpc^{-1}$ there is a 99.7% probability that the value Ω_M falls between 0.23 and 0.30. This means that assuming a flat universe ($\Omega_M+\Omega_\Lambda = 1$), the value of Ω_Λ should be between 0.77 and 0.70. Even if we change the Hubble constant, there the dark energy density parameter (Ω_Λ) will always be present, meaning that there has to be an energy field that is making the Universe accelerate.

*A prior is an approximation to an unknown o imprecise value in order to use it calculations. It can be fixed (using only one value) or marginalized (using various values).



Graph 7. Plot that shows the probability of each value of Ω_M from 0.15 to 0.40 assuming a flat universe and using *H_0=72 km s^{-1} Mpc^{-1}* as a prior for the blue line and *H_0= (70, 74) km s^{-1} Mpc^{-1}* as a marginalized prior for the red line. $\Delta\chi^2 = 1 = 1\sigma = 68.3\%$, $\Delta\chi^2 = 4 = 2\sigma = 95.4\%$, $\Delta\chi^2 = 9 = 3\sigma = 99.7\%$. Source: Own

3.3.1. Is it really Dark Energy?

But what if our measurements were false? It could be that the values for redshift (*z*) and distance modulus (*μ*) the Supernova Cosmology Project measured were caused by dust clouds that absorbed the light or maybe some unknown physics that stretches the wavelength of light. To see if dark energy could still exist if that was the case, we will have to make a likelihood analysis.

We know that the luminous flux (*F*, energy/ second/ unit of area) is defined by:

$$F = \frac{L}{4\pi r^2} \tag{41}$$

Where *L* is the luminosity in energy per second and the denominator is the area as a function of the radius (*r*).

Now we need to quantify the absorbance of the light. For this we can use an opacity parameter (τ) that will scale the luminous flux:

$$F_{obs} = F_{true} e^{-\tau} \tag{42}$$

Where F_{obs} is the observed luminous flux, F_{true} is the true or emitted luminous flux and τ is the opacity parameter. From there we can derive that:

$$D_L \propto e^{\frac{\tau}{2}} \tag{43}$$

Now we can see that if there are clouds or something else that is absorbing light, the opacity parameter (τ) will be bigger than zero. If $\tau > 0$, the luminosity distance (D_L) we will measure will be bigger than the real distance, since the object will appear darker to us.

If the opacity parameter (τ) is related to the luminosity distance (D_L), it also has to be a function of redshift (z). Assuming a linear dependence quantified by the epsilon parameter (ϵ) the function is:

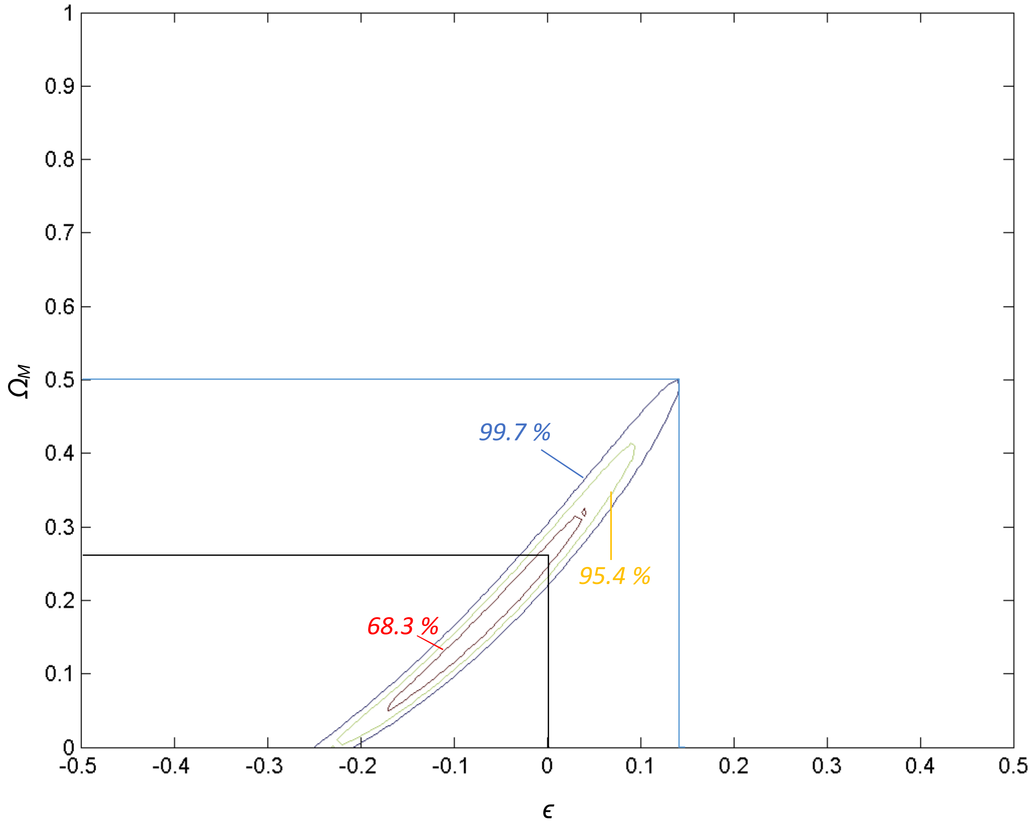
$$\tau(z) = 2\epsilon z \tag{44}$$

Following this assumption, the theoretical luminosity distance (D_{Lth}) we used in *Equation 40* now changes:

$$D_{Lth} \rightarrow D_{Lth} e^{\epsilon z} \tag{45}$$

Graph 8 was made by implementing this change and making a two-dimensional contour. A two-dimensional contour is a graph of two variables where the likelihood isn't shown as a variable. It substitutes the points we are used to seeing in graphs by a probability field where each point has a certain probability of being the correct one.

In *Graph 8*, we can see that if $\epsilon=0$ and, therefore, $\tau=0$ as well as there isn't any light absorption, the value we obtain for the matter density parameter (Ω_M) is probably the same we obtained earlier, around 0.27. Then, if there is light absorption, so that $\epsilon>0$, it is 99.7% sure that ϵ won't get bigger than 0.15. This means that the highest possible matter density (Ω_M) in our universe is around 0.50. Thus, with a flat universe ($\Omega_M+\Omega_\Lambda = 1$), in which $H_0=72\ km\ s^{-1}\ Mpc^{-1}$ and the highest possible light absorption, there still has to be a vacuum energy or dark energy present in the Universe.



Graph 8. Two-dimensional likelihood plot of the value of the matter density parameter (Ω_M) vs. the epsilon factor (ϵ) that is proportional to the opacity parameter (τ) assuming H_0=72 km s^{-1} Mpc^{-1}. At the red line $\Delta\chi^2 = 2.3 = 1\sigma = 68.3\%$, at the yellow line $\Delta\chi^2 = 6.12 = 2\sigma = 95.4\%$ and at the blue line $\Delta\chi^2 = 11.8 = 3\sigma = 99.7\%$. Source: Own

The negative values of epsilon, when $\epsilon<0$, would mean that the light gets brighter as it travels. This would require some new physics that creates light in the vacuum and, at the same time, it would mean that there is not a lot of matter in the Universe. However, we know that there is no physical law that allows the spontaneous creation of stable light in the vacuum. In addition, we know that Ω_M has to be at least bigger than 0.2 and it can't be 0, since we and everything we see is made out of matter.

Ruling out that ϵ could be negative, even if it gets to its maximum positive value ($\epsilon \approx 0.15$), we can see that the matter density parameter (Ω_M) can't be bigger than 0.5. Therefore, if we assume a flat universe ($\Omega_M + \Omega_\Lambda = 1$), the minimum value of Ω_Λ is 0.5, which means that there still has to be dark energy.

3.3.2. What is Dark Energy?

We have proven the existence of a vacuum energy that is making the expansion of the Universe accelerate. However, we don't know what it really is. So far, we have treated this vacuum energy or dark energy as if it were a constant. This so-called cosmological constant (Λ) is thought to be a field made of something unknown that has a uniform density everywhere in the Universe as we said in chapter 3.4.3.

It is not possible to find what makes up dark energy with the resources we have. However, we have the resources to try to prove what kind of density dark energy has and thereby find if it really is a constant. To do that we have to remember the equation of state (w_0) from chapter 3.3.2. and the proportionality that related it to the density (ρ) with the scale factor (a):

$$\rho \propto a^{-3(1+w_0)} \tag{46}$$

If we now write this as a function of redshift (z), we get:

$$\rho \propto a^{-3(1+w_0)} \propto (1+z)^{3(1+w_0)} \tag{47}$$

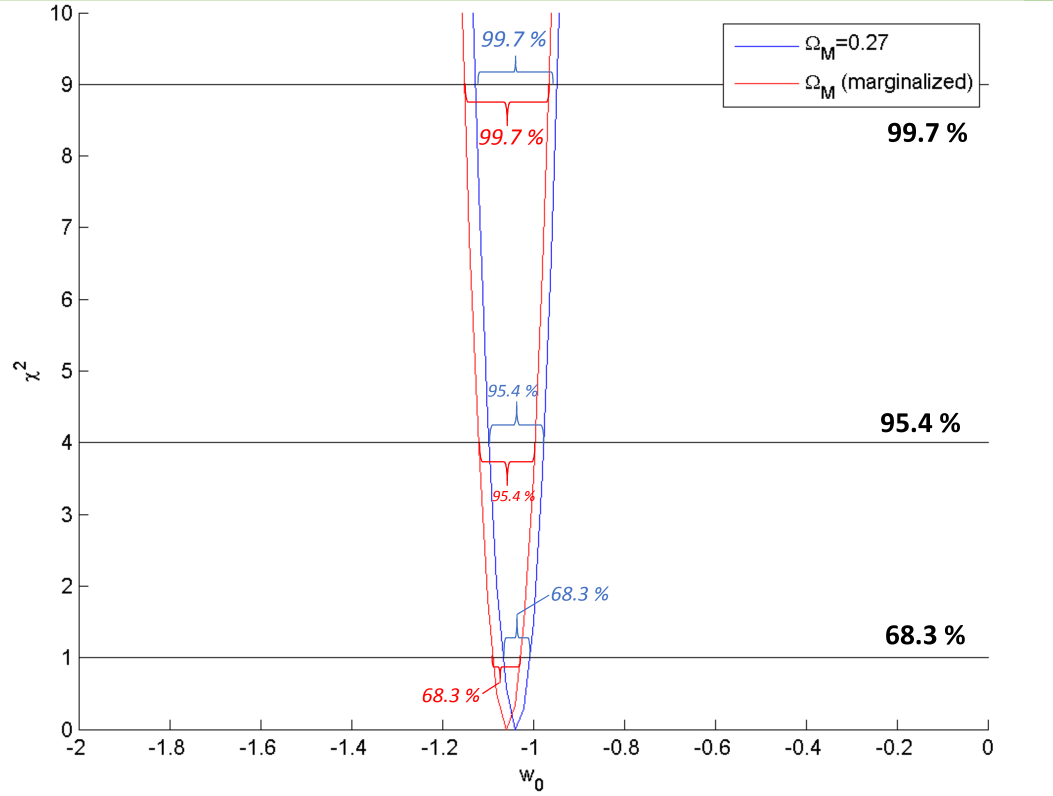
This allows us to introduce the equation of state as a constant into *Equation 37*, the Friedmann equation with density parameters:

$$H(z) = H_0\sqrt{\Omega_M(1+z)^3 + \Omega_D(1+z)^{3(1+w_0)}} \tag{48}$$

Where $H(z)$ is the Hubble parameter as a function of redshift (z), H_0 is the Hubble constant, Ω_M is the matter density parameter, Ω_D is the dark energy density parameter (since now we don't know it is a cosmological constant) and w_0 is the equation of state as a constant.

When we assumed that dark energy is a cosmological constant we were assuming that $w_0=-1$. We can see that if that happens, the whole $(1+z)^{3(1+w_0)}$ term from *Equation 48* cancels out and we are just left over with Ω_D or Ω_Λ. Now we don't assume that anymore and let w_0 vary too.

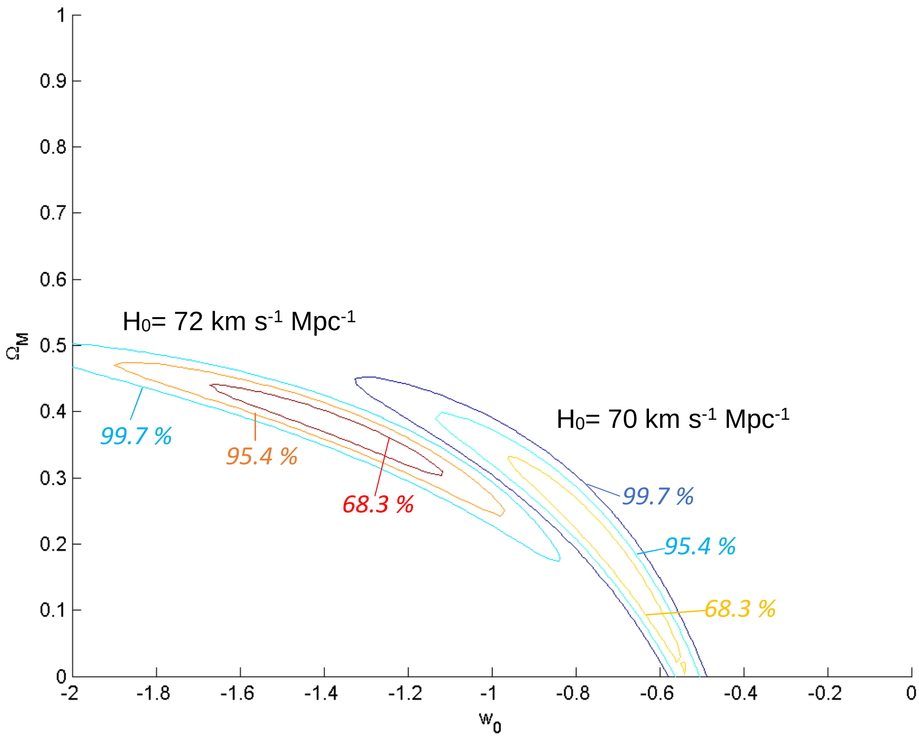
To get an idea of what might be the value of w_0, we need to find its likelihood by using *Equation 40* and *Equation 48*. Then we obtain *Graph 9*.



Graph 9. Plot that shows the probability for each value of the constant equation of state (w_0) where the blue line has a fixed prior for Ω_M at 0.27 and the red line has an equally weighted prior interval for Ω_M from 0.25 to 0.31. $\Delta\chi^2 = 1 = 1\sigma = 68.3\%$, $\Delta\chi^2 = 4 = 2\sigma = 95.4\%$, $\Delta\chi^2 = 9 = 3\sigma = 99.7\%$. Source: Own

In *Graph 9*, the likelihood of $w_0=-1$ when $\Omega_M=0.27$ and $H_0= 72\ km\ s^{-1}\ Mpc^{-1}$ is about 66.7 %. Nevertheless, this is a small probability using very fixed priors. In conclusion, this result isn't very valuable, because if we had used slightly different priors we could have obtained any result for w_0 we wanted.

In order to see what might be the constant equation of state (w_0) of dark energy when we change the priors, we will have to make a two-dimensional likelihood contour similar to the one we did for *Graph 8*. Nevertheless, there isn't any absorption now, but the density of dark energy can be different. Thus, the plot (*Graph 10*) shows the matter density parameter (Ω_M) vs. the constant equation of state (w_0).



Graph 10. Two-dimensional likelihood plot of the matter density parameter (Ω_M) vs. the constant equation of state (w_0) for two different H_0 priors. At the inner line $\Delta\chi^2 = 2.3 = 1\sigma = 68.3\%$, at the middle line $\Delta\chi^2 = 6.12 = 2\sigma = 95.4\%$ and at the outer line $\Delta\chi^2 = 11.8 = 3\sigma = 99.7\%$. Source: Own

Since the value of the constant equation of state (w_0) depends so much on the value of the Hubble constant (H_0), the uncertainty in the Hubble constant doesn't allow to extrapolate a value of w_0. This makes it hard to know what the behaviour of dark energy might be.

If w_0=-1, dark energy is a cosmological constant. If $w_0 \neq -1$, dark energy most certainly isn't a cosmological constant. Instead, it could be quintessence* (dynamic field) where some kinds have a varying equation of state and others a constant equation of state.

In order to have more precision we can add more data, but it wouldn't help much to add more supernovae data. Adding other types of data increases the precision even more due to its different behaviour. Completely different data can completely exclude values obtained from the other data that were still considered. For our purposes we will use a data set of Hubble parameter measurements from different sources (*Table 3*).

z	$H(z)$ (km s^{-1} Mpc^{-1})	σ_H (km s^{-1} Mpc^{-1})
0.070	69	19.6
0.100	69	12
0.120	68.6	26.2
0.170	83	8
0.179	75	4
0.199	75	5
0.200	72.9	29.6
0.270	77	14
0.280	88.8	36.6
0.350	76.3	5.6
0.352	83	14
0.400	95	17
0.440	82.6	7.8
0.480	97	62
0.593	104	13
0.600	87.9	6.1
0.680	92	8
0.730	97.3	7.0
0.781	105	12
0.875	125	17
0.880	90	40
0.900	117	23
1.037	154	20
1.300	168	17
1.430	177	18
1.530	140	14
1.750	202	40
2.300	224	8

Table 3. Data table of Hubble parameter ($H(z)$) measurements, with errors (σ_H), at different redshifts (z). Source: *Hubble Parameter Measure Constraints on the Cosmological Deceleration – Acceleration Transition Redshift* by Omar Farooq and Bharat Ratra

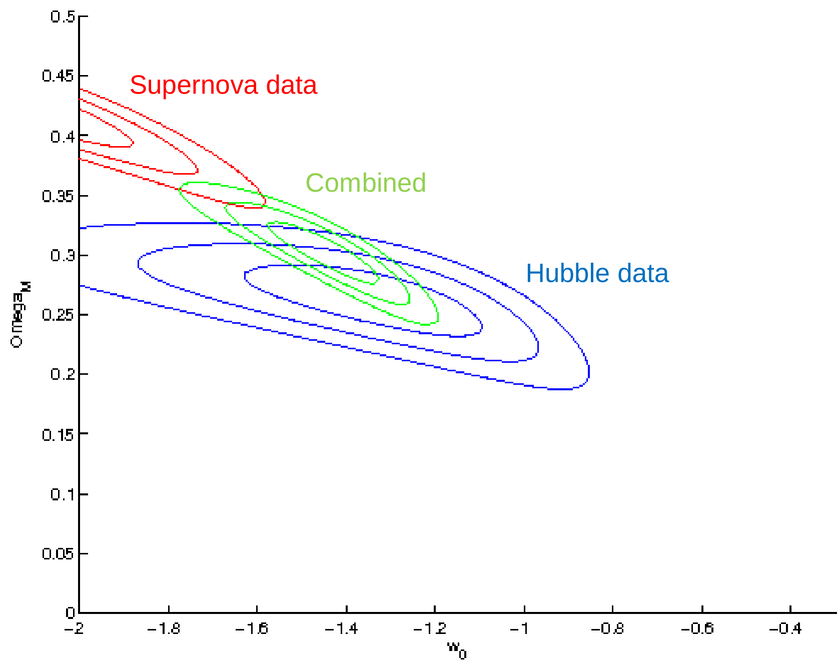
We do the same with this data as we did for *Graph 10*, but using this expression for the likelihood: (49)

$$\mathcal{L} = exp\left\{-\frac{1}{2}\sum_{i=1}^{28}\frac{[H_{obs}(i) - H_{th}(i,\Omega_M,w_0)]^2}{{\sigma_H}^2}\right\}$$

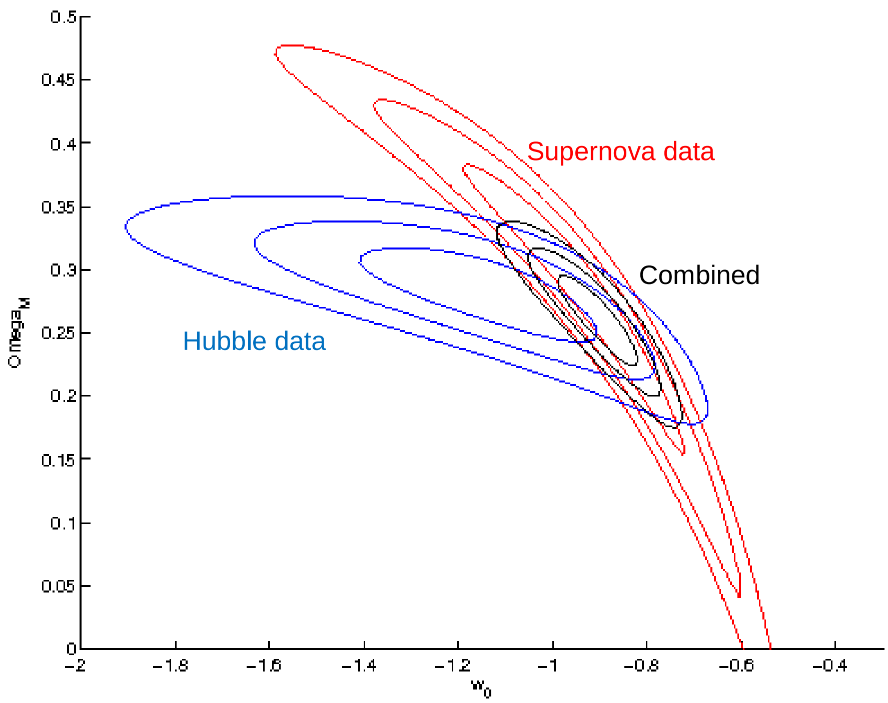
Where $\mathcal{L}$ is the likelihood, H_{obs} is the observed Hubble parameter, H_{th} is the predicted Hubble parameter, σ_H is the error of the observed Hubble parameter, Ω_M is the matter density parameter and w_0 is the constant equation of state.

Then we can apply *Equation 40* to obtain the X^2 likelihood. This likelihood is plotted together with the likelihood obtained from the supernovae data. In addition, the probability of those two datasets is combined by multiplying the probability of each point. From this we get a third two-dimensional contour that is smaller and more precise. *Graph 11*, *Graph 12* and *Graph 13* were made using the same data, but for each one the fixed H_0 prior was different, to show its effect on w_0.

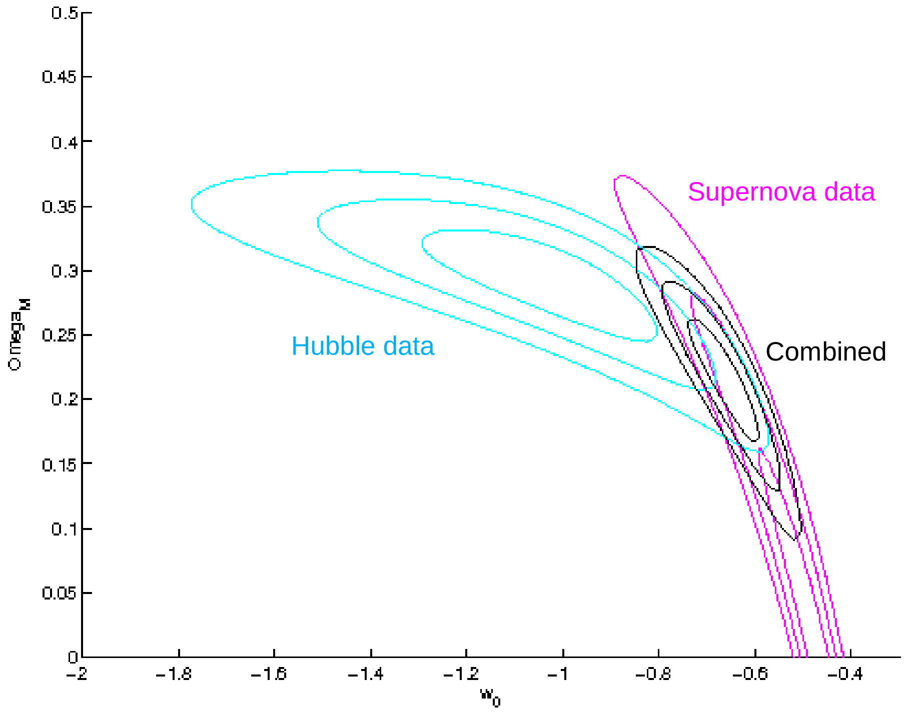
*Quintessence is a possible explanation for the essence of dark energy that describes it as the fifth fundamental force of nature. In this theory, quintessence is a field that varies over time and the force can be attractive or repulsive.



Graph 11. Two-dimensional likelihood plot of the Ω_M vs. w_0 assuming ***H_0=73.8 km s^{-1} Mpc^{-1}***. At the inner line $\Delta\chi^2 = 2.3 = 1\sigma = 68.3\%$, at the middle line $\Delta\chi^2 = 6.12 = 2\sigma = 95.4\%$ and at the outer line $\Delta\chi^2 = 11.8 = 3\sigma = 99.7\%$. Source: Own



Graph 12. Two-dimensional likelihood plot of the Ω_M vs. w_0 assuming ***H_0=70 km s^{-1} Mpc^{-1}***. At the inner line $\Delta\chi^2 = 2.3 = 1\sigma = 68.3\%$, at the middle line $\Delta\chi^2 = 6.12 = 2\sigma = 95.4\%$ and at the outer line $\Delta\chi^2 = 11.8 = 3\sigma = 99.7\%$. Source: Own



Graph 13. Two-dimensional likelihood plot of the Ω_M vs. w_0 assuming ***H_0=68.0 km s^{-1} Mpc^{-1}***. At the inner line $\Delta\chi^2 = 2.3 = 1\sigma = 68.3\%$, at the middle line $\Delta\chi^2 = 6.12 = 2\sigma = 95.4\%$ and at the outer line $\Delta\chi^2 = 11.8 = 3\sigma = 99.7\%$. Source: Own

By combining Hubble parameter measurement data with the Type 1a Supernovae data we were able to constrain the likelihood noticeably and to keep the value of Ω_M around 0.25, without surpassing 0.5. However, because of the strong effect the Hubble constant has on the Supernova data, it wasn't possible to get a clear result.

In conclusion, we have proven that it isn't incorrect to assume that dark energy is a cosmological constant, because it is still possible for it to be a cosmological constant. In order to get better results for the explanation of the essence of dark energy, we would need better Hubble constant (H_0) measurements, more data sets (like measurements of the fine-structure constant*) or try to test different models where w_0 isn't a constant. Even though the last two proposals could have been done, the objectives of this paper might not have been reached and too much unnecessary complexity would have been added.

So far, the essence of dark energy will remain a mystery, but in this instant scientists from all over the world are trying to solve it.

*The fine-structure constant is a dimensionless constant that describes the strength of the electromagnetic interaction and it is related to many of nature's constants. Some scientists suspect that it could be varying depending on where you are in the Universe.

4. CONCLUSIONS

4.1. Answers to the questions and hypothesis

The answers to the questions we postulated in the introduction of this project are:

- What is modern cosmology? Does it affect me?

  Modern cosmology is a young science that studies the origins, the evolution, the fate and the essence of our universe. It has been practised in a non-scientific manner for millennia, since those topics have always been interesting to humans.

  Modern cosmology is a science that affects us all. It can provide us with answers to some of our questions about the Universe. In addition, the search for those answers drives technological advancements that can have applications on our daily lives. However, some direct discoveries of modern cosmology might also have applications in engineering or other sciences. For example, without cosmology we wouldn't know about dark matter, which might someday have some practical applications.

- How does a modern cosmologist work?

  As we have seen in the theoretical part and practical part of this project, a modern cosmologist always follows the scientific method. In order to do that he has to propose models or theories to describe the Universe and then he or other cosmologists have to try to prove or disprove the model with observations.

  As we saw before, cosmological models aren't guesses about our universe anymore. They are mathematical and physical models built on previous theories, models and laws. To build those models a cosmologist requires a lot of mathematical skills and the ability to interpret maths physically.

  Once we have a model, this model has to be tested. To test those models the most common approach in cosmology is to use telescopes to observe far away or large-scale phenomena and see if they behave like it was predicted by the model. If they do behave as predicted the model is proven, until another experiment disproves it. Nevertheless, it is very difficult for cosmologists to reach certain conclusions because astronomical observations can sometimes be too imprecise (as we saw in 3.3.2.).

- Is our universe expanding? Is it accelerating?

  In the practical part of this project we concluded that the Universe is expanding, because we observe that the wavelength of the light emitted by far away objects gets stretched o redshifted. This can only happen if they are all moving away from us due to an expanding universe.

  We have also proven that our universe is accelerating, since we proved that the Hubble constant (H_0), the speed of the expansion of the Universe, changes over time. This change makes the Hubble constant increase over time and is therefore accelerating the expansion of the Universe.

- What makes our universe accelerate? What is it?

  If the Universe is accelerating, there has to be a force that is counteracting the attractive force of gravity. In the practical part of this project we have proven that most of the energy content in our universe is in the form of dark energy. This dark energy seems to have a repulsive force that is making the expansion of the Universe accelerate.

  We think that dark energy wasn't always accelerating the expansion of the Universe (see 2.3.4.b.). When matter density was higher than the dark energy density it was deescalating. However, we know that the current dark energy density is bigger than the current matter density. The most accepted explanation for that is that dark energy behaves like a cosmological constant (see 2.3.4.a.), which means that it always has the same density everywhere in space. This means that, even though initially matter was denser than dark energy, dark energy can be denser than matter now. Since the density of dark energy remained constant and the density of matter decreased as the Universe expanded in size. Thus, dark energy is most likely a field with the same density everywhere in the Universe. However, this is not certain yet and we still don't know what it is made of.

- What are the origins of our universe? How could its end be?

  As we saw in chapter 2.3.4.b., we currently think that our universe was created in the Big Bang. The Big Bang is the explosion that started the expansion of space from an infinitely small point.

  If the Universe is flat (see 2.3.3.) and dark energy truly has the same density everywhere in space, the expansion of the Universe is going to be accelerated as before, so it will get faster and faster. Someday the expansion of the Universe will be so fast that even our closest galaxies will move away from us faster than the speed of light. Then we won't be able to see any other galaxy anymore and only the stars from our galaxy, the Milky Way, will be visible. Then, if the expansion gets even faster, the expansion speed might get fast enough to overcome gravity. This will slowly pull galaxies apart and after a while, star systems like our Solar System. Eventually, the expansion speed will get so fast that even black holes and atoms get ripped apart. This continues until the whole Universe is nothing else than a soup of low energy photons that are separated by light years from each other. Nothing in the Universe will interact and entropy will be at is maximum.

  This only is the most accepted theory of many others. Depending on the true geometry of our universe and the nature of dark energy our universe could have different endings. Since we still don't know the exact geometry of our universe and the exact essence of dark energy, the exact fate of our universe will still remain unknown.

4.2. What have I learned?

This project gave me a deep understanding of modern cosmology and some of its topics. Most importantly it allowed me to improve many of my skills. I learned to work using the scientific method, my math skills improved, I learned to programme and my logical thinking improved too.

In addition, it was a great learning experience to write this project in English and Catalan. It was a great chance for me to practise both languages and to learn how to translate better. The translation of the entire project was a big effort that made me invest many more hours.

Finally, I think that this project was an important general learning experience that will be useful for any other projects or other pieces of work I will make in the future.

4.3. What did I like most/least?

The thing I like the most about doing this project is the physical interpretation of numbers. I found it very interesting to see some values or equations and from there being able to see what their effects on reality are. I also enjoyed doing the math to get to those equations.

I also liked to do the historical research of the origins of cosmology, since it was very interesting to see the evolution and to learn how some cultures thought the Universe works.

What I enjoyed the least was making the code. Not that I didn't like programming itself, but I found it really annoying when the code didn't work and I had to find the mistake in it. Thankfully, I had help while working on the code.

4.4. If I could start again, I…

If I could start again, I would do most things as I did them. Even though cosmology is a very complicated topic, I think I have given all the necessary information and organized it as good as possible. Maybe I would formulate some things differently and would consult even more sources to find some things I might have left out.

4.5. What did I leave out?

There were a few things that I had to leave out of this project. In the theoretical part, I only gave a brief explanation of some cosmological models there were in history. There are many more I couldn't talk about.

I also had to leave out many topics of modern cosmology that weren't relevant to the understanding of the science or the understanding of the practical part. Some of the topics I left out are nucleosynthesis (the creation of the first elements), the age of the Universe, neutrino cosmology, etc. I also didn't go very deep into dark matter, the inflationary model or the Big Bang theory.

In the practical part I could have continued adding more data or I could have tried to use other cosmological models than the ΛCDM models. This would have allowed me to get better results, but I think that this isn't too relevant for someone that is trying to understand modern cosmology and what it does. It is maybe more relevant to a cosmologist.

Finally, since modern cosmology is a science that is constantly evolving, this project will contain some ideas that will be different in a few years. Maybe some things I described will be disproven and other things I described as still uncertain will be certain. This is what makes this science so interesting.

5. DIARY

July 2014:

The 5th of July I went to Porto (Portugal) for two weeks to do a practicum at the *Centro de Astrofísica da Universidade do Porto* (CAUP) with the *Joves i Ciència* scholarship. There Prof. Dr. Carlos Martins tutored Catarina Rocha, Mar Pino and me. We learned about dark energy and worked on the code for 6-8 hours a day. We also attended a conference of Portuguese astronomers and astrophysicists.

During all the month of July I read the book *An Introduction to Modern Cosmology* by Andrew Liddle and a few scientific articles about dark energy, supernovae, etc., and continued working on the code. I was also continuously skyping with Carlos Martins, Catarina and Mar.

August 2014:

During the month of August I started planning the research project. I started talking to the school and chose my tutors. I also decided to make the project in two languages: English and Catalan. I wanted to make it in English, because most of my sources were in English. However, I also wanted to make it in Catalan, because it would make it easier to read for my teachers.

Since the practical part I proposed was quite an advanced one, Dolors (my tutor) and I came up with the idea to make a research project that targets all audiences by providing a simplistic explanation for every concept, but at the same time also providing a detailed mathematical description of each concept. We thought about dividing the complex part from the simple part, but we realized that this would only disrupt the continuity of the text. Therefore, we opted for making the mathematical explanation always follow the conceptual explanation of a topic. In addition, I continued working on the code as well as I continued reading scientific articles.

September 2014:

In September I searched for more sources, worked on the code and started developing the structure of the project. I didn't get to write much, since I was busier after school had started.

October 2014:

In October I wrote the first version of the table of contents and introduction of the research project. As I wrote it in English, I translated it to Catalan. Moreover, I started looking for historic sources about cosmology and started writing about the history of cosmology.

November 2014:

As I continued writing the theoretical part of the project, Dolors contacted the Prof. Àlvarez Gaumé (a cosmologist at CERN). He recommended us to contact Jaume Garriga and Roberto Emparan (two cosmologists at the University of Barcelona). I contacted them in December and they offered me to visit them after Christmas. Unfortunately, in the end we couldn't meet.

During the month of November I also finished parts of the code I wanted to include in this project.

December 2014:

In December I finished the theoretical part of this project. To do that I had to read some parts of *An Introduction to Modern Cosmology* again. I also had to read a few more scientific articles.

Usually, as I finished to write a chapter, I translated it to Catalan. I always forwarded the English part to Mercè and the Catalan part to Dolors. Then, when I got the corrections, I made the necessary changes, wrote a few more chapters and sent it to my tutors again.

Around Christmas, I started writing the practical part. Even though I already had the graphs and data tables since the summer. I still had to interpret them and introduce them into the project.

January 2015:

In the first week of January I finished the practical part in English and a few days later in Catalan. Then I wrote the conclusion and the bibliography. The last day before I had to hand the project in, I only made changes to the project and got corrections from my tutors.

6. ACKNOWLEDGEMENTS

I want to thank Prof. Dr. Carlos Martins for fascinating me for cosmology, giving me the idea for the practical part of this project and guiding me through it. I also want to thank Mar Pino and Catarina Rocha, the participants that programmed the code with me. I especially want to thank Maria Dolors Vidal Segarra and Mercè Izquierdo Zaragoza for their great support and for tutoring this project. Finally, I want to thank anyone who helped or supported me in this project.

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ENGLISH EDITION

Modern Cosmology for Everyone

Origin, Evolution and Fate of the Universe

ANNEXES

8. ANNEXES

8.1. Annex A: Supernovae Data Set

The “Union 2.1” compilation of type 1a supernovae data from the *Supernova Cosmology Project* from the Berkeley Lab (USA) is free use data base that contains data obtained from 2008 to 2011. It shows the name, the redshift (z), the distance module (μ) as well as the error of the distance module ($\Delta\mu$) of 580 type 1a supernovae.

To obtain the data, the *Supernova Cosmology Project* firstly observed large galaxy fields in order to find supernovae. Once some supernovae were found, a spectroscopy was made using the Keck telescope in Hawaii. With this spectroscopy, the type of supernovae could be determined. Then, the redshift and luminosity of the type 1a supernovae (the ones that always emit the same wavelength and brightness of light during a certain phase of their explosion) was analysed using telescopes like Hubble, Cerro Tololo (Chile), WIYN (USA) and Isaac Newton (Canary Islands). Because of this complex methodology, it is impossible for a high school student to obtain this kind of data.

More information about the *Supernova Cosmology Project* can be found on their website: *supernova.lbl.gov*

Supernova	z	μ	Δμ	
1993ah	0.028488	35.3465833928	0.223905932998	0.128418942246
1993ag	0.050043	36.6823679154	0.166828851413	0.128418942246
1993o	0.052926	36.8176912545	0.1557559148	0.128418942246
1993b	0.070086	37.4467365424	0.158466934433	0.128418942246
1992bs	0.062668	37.4834093505	0.156099434739	0.128418942246
1992br	0.087589	38.2290570494	0.187745679272	0.128418942246
1992bp	0.078577	37.4881622607	0.155635656185	0.128418942246
1992bo	0.017227	34.6543699503	0.199337179559	0.128418942246
1992bl	0.042233	36.3364595483	0.167174042338	0.128418942246
1992bh	0.045295	36.6402721756	0.164981248644	0.128418942246
1992bg	0.03648	35.9053219652	0.170174952845	0.128418942246
1992bc	0.019599	34.5852174312	0.184691219687	0.128418942246
1992aq	0.100915	38.4567455954	0.167333481677	0.128418942246
1992ag	0.027342	35.085765693	0.175510835947	0.128418942246
1992ae	0.074605	37.5881157565	0.15977086456	0.128418942246
1992p	0.026489	35.4806851993	0.19131226974	0.001870261557
1990af	0.049922	36.5669734706	0.162303819627	0.128418942246
1990o	0.030604	35.5502377594	0.173295444142	0.128418942246
2001cz	0.016345641	34.0440277752	0.142912931364	0.128418942246
2001cn	0.0154363	33.9409483971	0.148694109648	0.128418942246
2001ba	0.030529	35.5992457186	0.088750663709	0.128418942246
2000ca	0.024525	35.0581706649	0.102438504345	0.128418942246
2000bh	0.023953	34.9687103775	0.107041197195	0.128418942246
1999gp	0.026038	35.3672620682	0.108499792159	0.128418942246
1993ac	0.048948	36.7315973987	0.172547618731	0.0
1994m	0.024314	35.1094950645	0.181662706307	6.1999960721-11
1994s	0.015166	34.1016666152	0.215239340732	0.009354993721
1994t	0.03572	35.9605406407	0.171186987363	0.79821525066
1995ac	0.048818	36.3820107761	0.160299265066	5.14460000156-08
1995ak	0.0219800059146	34.8529733574	0.187544764358	0.128418942246
1996ab	0.1244	39.0447885112	0.164268687505	0.128418942246
1996bl	0.036	35.8210170957	0.167885249541	0.128418942246
1996bo	0.016321	34.0174211141	0.204965074268	0.128418942246
1996bv	0.01673	34.2263371692	0.209464670311	0.128418942246
1996c	0.0275	35.6497059082	0.176364688958	0.218251065653
2000fa	0.021793	34.9737868712	0.232449382954	0.128418942246
2000dk	0.01645	34.1812962858	0.250896560248	0.128418942246
2000cn	0.023208	35.0855427174	0.231479604833	0.128418942246
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1998eg	0.023536	35.1695990888	0.234698201629	0.128418942246
1998ef	0.016743	34.0027278026	0.248385125526	0.128418942246
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1998co	0.016991	34.3787718073	0.309889810973	0.128418942246
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1998v	0.017173	34.2606714573	0.248216009365	0.128418942246
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2005mc	0.026	35.3812932276	0.183933665117	1.7670300001-07
2005ms	0.0259	35.4159686901	0.178489658823	0.033789980906
2006ac	0.0239	35.0349741436	0.181392417208	1.09587499997-06
2006al	0.069	37.566047978	0.176163639895	3.07778750001-05
2006an	0.0651	37.3066907976	0.162808584338	1.0
2006ar	0.0229	35.1968671677	0.185459075264	0.089659819877
2006az	0.0315	35.651136343	0.170993523745	0.0
2006bq	0.0215	34.9327363282	0.187040817965	1.1733000016-08
2006br	0.0255	35.7201731359	0.196167302112	0.128418942246
2006bt	0.0325	35.8130914151	0.170394735896	0.0
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2006bw	0.0308	35.6289808082	0.178370444318	0.128418942246
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2006cj	0.0684	37.7311969845	0.170591643132	0.0
2006cm	0.0153	34.7071857289	0.213242896074	0.128418942246
2006cp	0.0233	34.8821286745	0.183424685088	0.011101109848
2006cq	0.0491	36.7301358821	0.175843223164	0.0
2006cz	0.0425	35.9281018139	0.195296013078	0.128418942246
2006ej	0.0192	34.7366784514	0.197842881265	8.36000046966-10
2006en	0.0308	35.7791362444	0.173310004596	0.128418942246
2006et	0.0212	34.8471345313	0.19152972211	0.128418942246
2006gj	0.0277	35.7050037726	0.183298735051	0.128418942246
2006gr	0.0335	35.9738325393	0.170490251613	0.128418942246
2006kf	0.0208	34.7954563657	0.201629405401	0.128418942246
2006le	0.0173	34.2300660446	0.220994474003	0.128418942246
2006mo	0.036	36.146298861	0.177000363965	0.128418942246
2006mp	0.0233	35.1983574195	0.185738575456	0.128418942246
2006oa	0.0589	37.1116475131	0.16568235078	1.0
2006ob	0.0583	37.0582881494	0.16808968138	0.0
2006on	0.0688	37.4866356466	0.198025808769	0.027111473488
2006os	0.0321	35.6480768335	0.175025422739	0.128418942246
2006ot	0.0522	36.6743146094	0.189117803003	0.128418942246
2006qo	0.0308	35.5935042223	0.172493084423	0.128418942246
2006s	0.0329	35.9417685428	0.170393175896	0.00076391263
2006sr	0.023	35.0706425048	0.186848384979	0.128418942246
2006td	0.015	34.3797728047	0.217348534701	0.128418942246
2006te	0.0321	35.8703618342	0.1737410077	0.000101920284
2007ae	0.0643	37.1755022179	0.166008010074	0.128418942246
2007ai	0.032	35.8266913419	0.193104691662	0.128418942246
2007au	0.0209	34.7028984995	0.19422693515	0.128418942246
2007bc	0.0219	34.8489904808	0.188387184264	3.80834000002-07
2007bd	0.032	35.5884973241	0.173446310936	7.81762000024-07
2007ca	0.0151	34.5258752881	0.213682160531	0.128418942246
2007ci	0.0192	34.4915623332	0.197091647943	0.006396224841
2007co	0.0266	35.3234935897	0.177716836428	0.128418942246
2007cp	0.0377	35.7952144325	0.294106560612	0.128418942246
2007cq	0.0247	34.9136928277	0.181621011598	0.128418942246
2007f	0.0242	35.1893805829	0.180174916343	0.435402819564
2007o	0.0366	35.9712906539	0.168963687947	0.128418942246
2007qe	0.0229	35.1386260702	0.183018039949	0.128418942246
2007r	0.0312	35.8828585622	0.173762463024	0.0
2007s	0.015	34.1114109643	0.213388869977	0.128418942246

2008af	0.0341	35.770474764	0.177569140796	0.0
2008bf	0.0251	34.9482752367	0.179222700336	0.0
2008l	0.0189	34.3747016599	0.209571630196	0.10474237468
2004ef	0.029802136901	35.4709217104	0.122226817874	0.128418942246
2004gc	0.0321340168614	35.3781278853	0.127512090116	0.128418942246
2004gs	0.0275687259109	35.4754718132	0.128117983088	0.128418942246
2004gu	0.0469673346156	36.4944134465	0.110309142172	0.128418942246
2005a	0.0183152320362	34.3717548996	0.160464926406	0.128418942246
2005ag	0.0800481440021	37.6857093353	0.101968492144	0.128418942246
2005bg	0.0241852990802	35.052324275	0.133368622118	0.128418942246
2005bo	0.0150270427632	33.9501915023	0.178117024083	0.128418942246
2005eq	0.0283960268116	35.5481484694	0.125198881719	9.90000303958-11
2005hc	0.0449766726113	36.5503954158	0.108342547225	0.930467219184
2005iq	0.0329123710881	35.9678520173	0.118114324121	0.128418942246
2005ir	0.0753501119815	37.5800899168	0.102274672969	0.996080571775
2005ki	0.0203747245299	34.6564438473	0.145603349944	0.000769867316
2005m	0.0229711675782	35.133882951	0.135640926041	0.128418942246
2005na	0.0268091974805	35.2601505395	0.127879595703	0.128418942246
2006ax	0.0179312833626	34.3498200027	0.155413940574	0.128418942246
2006eq	0.0483921953954	36.6858343824	0.119215628214	0.128418942246
2006py	0.0566833670752	36.9636230671	0.110070937939	0.128418942246
2005kt	0.0638640836964	37.3159416121	0.119614965162	0.551671793295
10106	0.146290296068	39.5590095488	0.120248735474	0.551671793295
2005ez	0.129278206634	38.9180009489	0.134402615626	2.58499999184-09
2005lk	0.102715033935	38.4872595888	0.119892508421	0.551671793295
2005ll	0.242504680476	40.1304465537	0.150915056663	0.551671793295
2005lf	0.29840927363	41.0624002336	0.21581859647	0.551671793295
2005ku	0.0437189114895	36.3865660158	0.127214292529	0.551671793295
2005ml	0.113042644637	38.5514541276	0.119237437717	0.551671793295
2005jb	0.256475742725	40.6108439003	0.156286225391	0.551671793295
2005fc	0.295585550063	41.1370597018	0.215064876219	0.551671793295
2005mm	0.380359487336	41.657498659	0.217910527631	0.551671793295
2005ln	0.145668547261	39.056715703	0.125750126249	0.551671793295
2005mo	0.273454768884	40.7271582441	0.163668270948	0.551671793295
2005lo	0.297518833561	40.7722050136	0.205185826917	0.551671793295
2005lq	0.378965802039	41.5814889023	0.197094007682	0.551671793295
1166	0.380416514286	41.2723405299	0.198558434394	0.551671793295
2005lp	0.301755029535	41.4938104959	0.246761480943	0.551671793295
2005mq	0.348345020835	41.3077915312	0.217774889061	0.551671793295
2005ff	0.0856894593607	37.9974134061	0.116365280048	0.004916236676
2005fd	0.260586108317	40.5231676777	0.141595051803	0.551671793295
2005fe	0.215543320953	40.2965294236	0.140207657342	0.551671793295
2005fh	0.117625328764	38.5794185081	0.113609962836	7.41867000054-07
2005gj	0.18221823956	39.5938188438	0.119334463704	0.551671793295
1688	0.357507357032	41.3288394045	0.202587682493	0.551671793295
2005fj	0.141787999126	39.2738113329	0.119521982693	0.551671793295
2005fo	0.260533476953	40.8081341413	0.148279348857	0.551671793295
2005fl	0.232781107182	40.2045290903	0.15178836784	0.551671793295
2005fm	0.151857894665	39.155788359	0.113847311938	0.989888621753
2005fn	0.0939086318905	38.1728205141	0.117952906116	0.551671793295
2005fr	0.286618706522	41.0384076468	0.133800275646	0.551671793295
2005fy	0.194316512142	39.9615200415	0.126373844923	0.551671793295
2005ey	0.147025138268	39.3035839668	0.111657296738	4.30000479668-11
2005fp	0.2115869815	40.5603236924	0.159324733885	0.551671793295
2005ft	0.180119779968	39.6385777035	0.115560254718	0.551671793295
2005fi	0.263491026592	40.7673226085	0.127293080139	0.551671793295
2005fu	0.192149980044	40.0392178782	0.121087443202	0.001229231543
2005fs	0.338802609134	41.3053718801	0.170908367946	0.551671793295
2005fv	0.117277362889	38.7459338248	0.113702596154	0.551671793295
2005fw	0.1424046524	39.1164772511	0.113661193986	0.830402042657
2005fa	0.160861854636	39.3191758892	0.11864489981	9.9997787828-13
2005fx	0.288418344585	40.8427853587	0.149598782119	0.551671793295
2005fz	0.12282889997	38.7997897008	0.12106543913	0.551671793295
2005go	0.263647950642	40.5481756685	0.139823620366	0.551671793295
2005gp	0.126473161901	38.7104691761	0.119200794649	0.586055835562
2005ga	0.172742230556	39.5029361421	0.113458015916	0.551671793295
2005gc	0.163795894987	39.3945630608	0.113134872307	1.0
2005gs	0.249511054792	40.7809243328	0.132323047466	0.551671793295
2005gh	0.257740303509	40.6514783957	0.135891055781	0.551671793295
2005hn	0.106712340103	38.6366669013	0.117621429285	0.867576933443
2005gd	0.159889937748	39.3505845262	0.114747057769	0.999968604262
2005ge	0.204979685349	40.0245002346	0.123526942937	0.551671793295
2005gr	0.244378876797	40.2019303938	0.124309900698	0.551671793295
2005gf	0.248508130676	40.2721746838	0.125645589328	0.551671793295
2005gg	0.228528473881	40.2544336386	0.12201137132	0.551671793295

2005gb	0.0858546441437	37.9524441212	0.112589853288	1.0
2005ho	0.0618357646938	37.1309944806	0.115680071342	0.987710988495
2005gt	0.277853422619	40.8380084515	0.168994207516	0.551671793295
2005gw	0.275440197435	40.743391802	0.142941615821	0.551671793295
4064	0.155247328221	39.3173204059	0.115415755865	0.551671793295
2005gu	0.330512448683	41.2279390018	0.156723683034	0.551671793295
2005gv	0.361934308832	41.3844533517	0.166441914984	0.551671793295
2005gy	0.33063462833	41.0522598891	0.152699339448	0.551671793295
2005gx	0.144621085972	39.290829773	0.112727024487	1.0
2005gq	0.389288787688	41.6026284289	0.200867575788	0.551671793295
2005hp	0.173910056566	39.5319411053	0.128967775815	0.551671793295
2005hs	0.300312695802	40.8468481928	0.15150098613	0.551671793295
2005hr	0.116348502599	38.7461381651	0.112296117135	1.0
2005hu	0.218585189041	40.1990974567	0.117113950953	0.551671793295
2005hx	0.119671538006	38.7555381178	0.110740033634	0.551671793295
2005hy	0.154632097239	39.3239502295	0.112047393667	0.999997454964
2005hw	0.408319092164	41.840293526	0.206369618511	0.551671793295
2005hv	0.177600694977	40.070675851	0.12481415503	1.0
2005ia	0.25066763111	40.7456030264	0.120970959661	0.551671793295
2005jz	0.251740186424	40.5417217599	0.118440110682	0.551671793295
5737	0.391599213441	41.5087271022	0.171564315895	0.551671793295
2005hz	0.128726735467	38.8642253068	0.109513340257	0.551671793295
2005hq	0.399601334976	41.7288003127	0.205132433748	0.551671793295
2005ic	0.309492645411	41.0181924732	0.134960171498	0.551671793295
2005is	0.170628396749	39.474900116	0.116376625267	0.551671793295
2005ie	0.278924676328	40.7678297655	0.130157044183	0.551671793295
2005it	0.308580866071	40.9076217053	0.177652782589	0.551671793295
2005ht	0.18581244671	39.8240975844	0.120667757481	0.551671793295
2005if	0.0664403123402	37.3721050548	0.1175824014	0.543563084394
2005ka	0.316429844695	41.3950733806	0.192866122248	0.551671793295
2005ih	0.257497888297	40.5941645935	0.131393591319	0.551671793295
2005iv	0.298777444088	41.0293579818	0.154536935815	0.551671793295
2005jy	0.270434442827	40.668983836	0.145477928824	0.551671793295
2005ig	0.27945473261	40.5012843665	0.138724732909	0.551671793295
2005ii	0.292469756082	40.919048654	0.141400626542	0.551671793295
2005jk	0.188853175387	39.7920374862	0.119741860505	7.8208394-05
2005ix	0.265762479518	40.4981499217	0.124803942242	0.551671793295
2005ij	0.124273528772	38.719986901	0.111134858876	0.029911743888
2005id	0.182548913397	39.7840215634	0.114083136974	0.986226043013
2005jd	0.312883364158	41.0144487771	0.139506514174	0.551671793295
2005ik	0.309547337158	41.1925567238	0.154012332803	0.551671793295
2005iu	0.0890194292939	37.8284529286	0.117432705446	0.551671793295
2005iy	0.402459619216	41.9111467578	0.211278493994	0.551671793295
2005iz	0.200611719928	39.8523918219	0.132867364384	1.0
2005ja	0.326396483203	40.972304176	0.144388065833	0.551671793295
2005jc	0.211629598493	39.9801047509	0.115235180715	0.551671793295
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2005je	0.0931494026855	38.280913725	0.114453830019	0.551671793295
2005jg	0.30240162108	41.033200098	0.134910116135	0.551671793295
2005jh	0.108638265524	38.6505001979	0.119489654857	0.551671793295
2005ed	0.0856961170652	37.9937917107	0.117971237872	0.551671793295
2005jm	0.202608679897	39.9677592554	0.128807331127	1.0
2005kn	0.196716068579	39.9244173787	0.134983153598	0.551671793295
2005ei	0.12668798909	38.9152084322	0.123937308071	0.551671793295
2005ji	0.214568258658	40.2018496659	0.11698744578	0.551671793295
2005jn	0.320446988861	41.102785135	0.134380311167	0.551671793295
2005jo	0.218347445403	40.2208498377	0.124421502953	0.551671793295
2005eg	0.189706566951	39.9395625109	0.119682792229	0.551671793295
2005ex	0.0929368181997	38.1458300404	0.118950355954	0.551671793295
2005jw	0.379662293575	41.4330618932	0.147219740299	0.551671793295
2005jp	0.210938395061	39.983937734	0.120818770577	0.551671793295
2005jj	0.366602898789	41.8598572351	0.198404429556	0.551671793295
2005jv	0.420926821103	42.1335480049	0.202321001311	0.551671793295
2005ju	0.258028270562	40.6446788667	0.137108498782	0.551671793295
2005ko	0.183568404877	39.7107958033	0.123775734742	0.551671793295
2005mi	0.212548765094	40.0875932773	0.13157696281	0.551671793295
2005jt	0.360034211989	41.1573004359	0.175048186524	0.551671793295
2005mh	0.393974478272	41.7811534223	0.150707319622	0.551671793295
2005kp	0.11471262087	38.669095116	0.113372328428	1.0
2005ld	0.143705907	39.1857706544	0.116311967547	0.372920925377
2005le	0.252486060029	40.5184210801	0.145050195669	0.551671793295
2005kq	0.387297107465	41.8924653292	0.18642986619	0.551671793295
2005lg	0.348583857544	41.3009410359	0.177371887871	0.551671793295
2005li	0.255490620022	40.4859446261	0.150967373278	0.551671793295
2005lh	0.216582821954	40.3602717031	0.152479838933	0.551671793295

1996e	0.43	41.3188581715	0.357827626062	0.551671793295
1996h	0.62	43.2279620233	0.390344663066	0.551671793295
1996i	0.57	42.4891103591	0.388952633173	0.551671793295
1996j	0.3	40.9627951522	0.314865571912	0.551671793295
1996k	0.38	42.063176534	0.327833114422	0.551671793295
1996u	0.43	42.3985415069	0.458327873977	0.551671793295
1995ao	0.24	40.7421915815	0.402844159873	0.551671793295
1997ce	0.44	42.0495332736	0.319794878908	0.551671793295
1997cj	0.5	42.3630032623	0.317337166801	0.551671793295
1997ck	0.97	42.8212038325	0.805210302782	0.551671793295
1995k	0.479	42.3517577625	0.358136299792	0.551671793295
1997ap	0.83	43.5424034366	0.471046842772	0.551671793295
1997am	0.416	42.4236170407	0.556342333092	0.551671793295
1997aj	0.581	42.063423278	0.505033960368	0.551671793295
1997ai	0.45	41.8319918495	0.453701962845	0.551671793295
1997af	0.579	43.1863633442	0.643297806625	0.551671793295
1997ac	0.32	41.2446034516	0.420998113276	0.551671793295
1997r	0.657	42.9752487173	0.658728481435	0.551671793295
1997q	0.43	41.7661368295	0.608328916424	0.551671793295
1997p	0.472	41.9691632564	0.514472850425	0.551671793295
1997o	0.374	43.1851146804	0.923198713714	0.551671793295
1997n	0.18	40.1895173161	0.445447560867	0.551671793295
1997l	0.55	44.3434675429	1.00681218041	0.551671793295
1997k	0.592	44.1523548256	0.717963493004	0.551671793295
1997i	0.172	39.3022260063	0.429709120635	0.551671793295
1997h	0.526	41.9560808598	0.513679185483	0.551671793295
1997g	0.763	44.4724245942	0.898163121417	0.551671793295
1997f	0.58	43.3051250139	0.519962251252	0.551671793295
1996cn	0.43	41.8019360973	0.458804990654	0.551671793295
1996cm	0.45	42.2709731768	0.589947886789	0.551671793295
1996ck	0.656	43.1448591059	0.627202505766	0.551671793295
1996ci	0.495	42.1189081538	0.443653473108	0.551671793295
1996cg	0.49	41.7874349383	0.446484505947	0.551671793295
1996cf	0.57	42.6714667124	0.470382088569	0.551671793295
1995ba	0.388	42.2076140366	0.465017827284	0.551671793295
1995az	0.45	42.4024477254	0.508290617768	0.551671793295
1995ay	0.48	42.1599485926	0.521420422571	0.551671793295
1995ax	0.615	42.542636608	0.556222743429	0.551671793295
1995aw	0.4	42.3127671586	0.477589020966	0.551671793295
1995at	0.655	42.3150852482	0.497081624743	0.551671793295
1995as	0.498	42.9837202371	0.642406504117	0.551671793295
1995ar	0.465	41.8251633017	0.600030636232	0.551671793295
1995aq	0.453	42.8212084765	0.524912951846	0.551671793295
1994g	0.425	41.2021312228	0.489149082777	0.551671793295
2002ad	0.514	42.7861936213	0.493735911043	0.551671793295
2002ab	0.423	41.5655258816	0.243579000627	0.551671793295
2002x	0.859	44.0929026019	0.296965045086	0.551671793295
2001kd	0.936	43.3071058635	0.705981399741	0.551671793295
2001jp	0.528	42.4543495693	0.24429067529	0.551671793295
2001jm	0.978	43.5000721647	0.292647301809	0.551671793295
2001jh	0.885	44.1843333063	0.286230184051	0.551671793295
2001jf	0.815	44.0744016007	0.728921730286	0.551671793295
2001jb	0.698	43.768728686	0.433480217975	0.551671793295
2001iy	0.568	42.7092496229	0.289546896784	0.551671793295
2001ix	0.711	43.5665495646	0.366725181041	0.551671793295
2001iw	0.3396	41.0839733838	0.226527941211	0.551671793295
2001iv	0.3965	41.4879207897	0.202374999057	0.551671793295
2001hy	0.812	43.6535732762	0.379016341937	0.551671793295
2001hx	0.799	43.3788294006	0.233174228134	0.551671793295
2001hu	0.882	43.3750389271	0.583987119465	0.551671793295
2001hs	0.833	43.6880978478	0.520913245917	0.551671793295
2001fs	0.874	43.295379241	0.388089303157	0.551671793295
2001fo	0.772	43.5099938279	0.218834770025	0.551671793295
1999du	0.178	39.454790821	0.235746048095	0.551671793295
1999dr	0.26	40.8233657709	0.199851735971	0.551671793295
1999du	0.186	39.7121679254	0.189043139487	0.551671793295
1999dx	0.269	40.7761617315	0.25640063853	0.551671793295
199dy	0.215	40.3780576732	0.193933997767	0.551671793295
2000fr	0.543	42.4788058506	0.0981414342081	0.551671793295
1998bi	0.75	43.242998687	0.138318457637	0.551671793295
1998be	0.64	42.7642472749	0.18880937743	0.551671793295
1998ba	0.43	42.1841242053	0.148171058429	0.551671793295
1998ay	0.64	43.1643806938	0.193749016644	0.551671793295
1998ax	0.497	42.3245530903	0.167980194286	0.551671793295
1998aw	0.44	42.0106475331	0.107484807666	0.551671793295

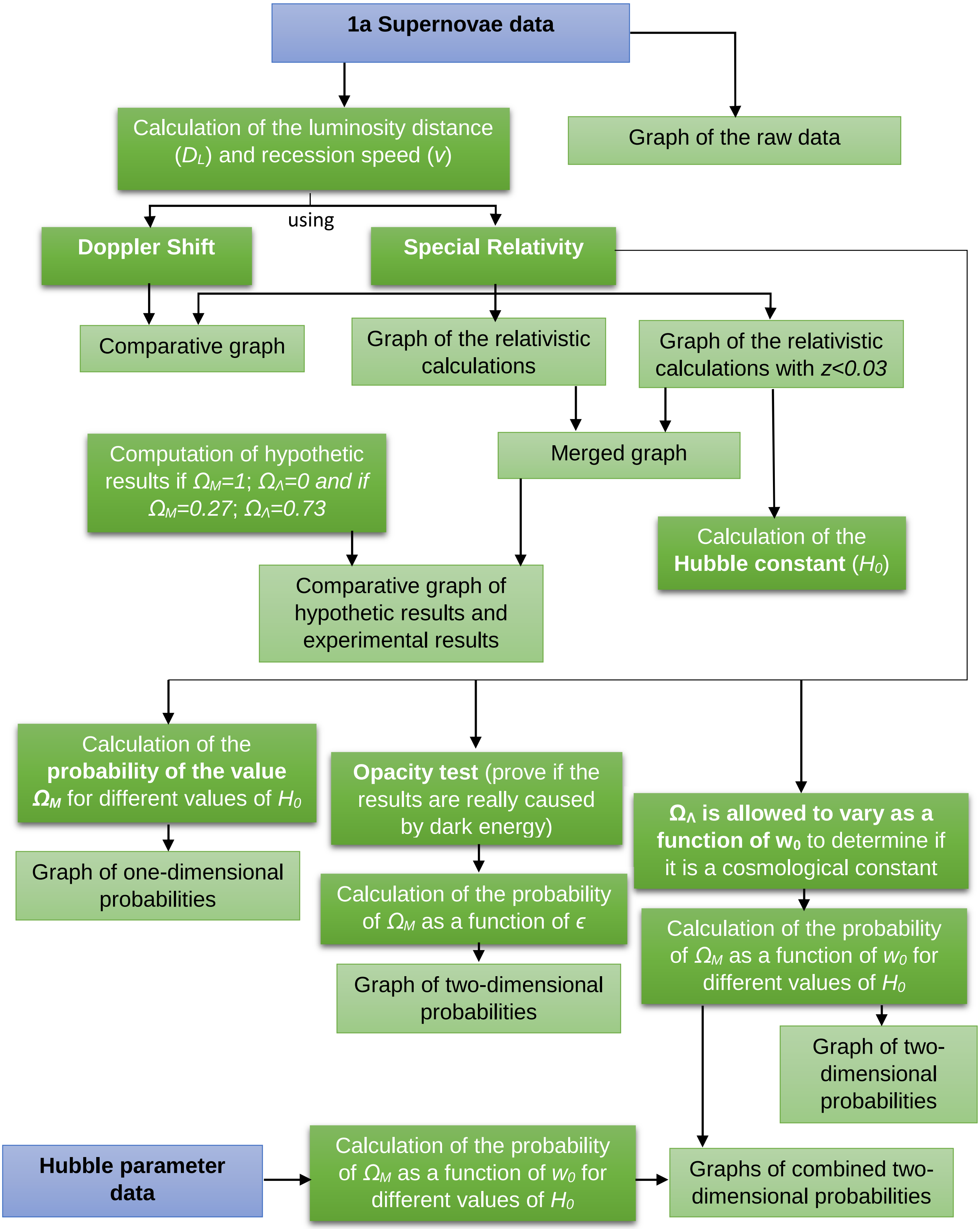
1998as	**0.355**	**41.3423115755**	**0.20146843402**	**0.551671793295**
1997ez	**0.78**	**43.6008071703**	**0.171318833305**	**0.551671793295**
1997eq	**0.54**	**42.4246212672**	**0.111551880919**	**0.551671793295**
1997ek	**0.86**	**43.9216606703**	**0.172411965167**	**0.551671793295**
03D4au	**0.468**	**42.5489037371**	**0.1639147706**	**0.551671793295**
04D4bk	**0.84**	**43.8731031943**	**0.222348502282**	**0.910256228198**
04D3nr	**0.96**	**43.6132116082**	**0.275514631381**	**0.551671793295**
04D3lu	**0.8218**	**43.8153639621**	**0.21320698395**	**0.0**
04D3ki	**0.93**	**43.5524295782**	**0.289918379066**	**0.551671793295**
04D3gt	**0.451**	**41.7966744145**	**0.143055821894**	**0.551671793295**
04D3do	**0.61**	**42.904965712**	**0.156248253322**	**0.0**
04D3cp	**0.83**	**44.0506599335**	**0.208835355956**	**0.019019116792**
04D2gp	**0.707**	**43.2824256201**	**0.256253916455**	**0.999999011725**
04D2fp	**0.415**	**41.8767704643**	**0.141093702805**	**0.125506643627**
04D1ag	**0.557**	**42.5570112027**	**0.157456031152**	**0.551671793295**
03D4fd	**0.791**	**43.5768811197**	**0.201343679161**	**0.424378352473**
03D4cz	**0.695**	**43.2117042607**	**0.20544919439**	**0.048213467444**
03D4at	**0.633**	**43.0498635302**	**0.169957137727**	**1.0**
03D3bh	**0.2486**	**40.6111853429**	**0.154515955385**	**0.551671793295**
03D3af	**0.532**	**42.5658156596**	**0.166651053217**	**0.551671793295**
03D1fc	**0.331**	**41.0784700501**	**0.137704766569**	**0.0**
03D1bp	**0.346**	**41.3646268615**	**0.15221324645**	**9.00002294912-12**
04D4dw	**0.961**	**44.2642016168**	**0.340056560686**	**9.6749703-05**
04D4an	**0.613**	**42.9937246636**	**0.165096720419**	**0.000122102116**
04D3nh	**0.3402**	**41.3280915278**	**0.135546197344**	**1.0**
04D3lp	**0.983**	**44.1572843671**	**0.43499764107**	**1.0**
04D3is	**0.71**	**43.0220388098**	**0.184280954678**	**0.551671793295**
04D3fq	**0.73**	**43.27699552**	**0.194894052413**	**1.0**
04D3df	**0.47**	**42.1345594858**	**0.15323960172**	**0.551671793295**
04D3co	**0.62**	**43.0092526047**	**0.169564685824**	**0.964610252373**
04D2gc	**0.521**	**42.1801294521**	**0.160996803083**	**0.998949555959**
04D2cf	**0.369**	**41.6340230154**	**0.152644121885**	**7.25190000272-08**
03D4gl	**0.571**	**42.3991620661**	**0.174223963618**	**0.999999999363**
03D4dy	**0.604**	**42.526976481**	**0.15968704969**	**0.991658804539**
03D4cy	**0.9271**	**43.9404978712**	**0.286707554506**	**0.846559054956**
03D4ag	**0.285**	**40.8542434728**	**0.136594130276**	**5.69999603073-11**
03D3ba	**0.2912**	**40.8390642775**	**0.146624503178**	**0.005691513851**
03D1gt	**0.548**	**42.2962985563**	**0.18837089341**	**0.551671793295**
03D1ew	**0.868**	**43.4932874061**	**0.24609516526**	**0.921233644992**
03D1ax	**0.496**	**42.2146760288**	**0.152814539841**	**0.0**
04D4dm	**0.811**	**43.4045360216**	**0.210619526441**	**1.0**
04D3oe	**0.756**	**43.8139921904**	**0.192775955331**	**0.551671793295**
04D3nc	**0.817**	**43.6515637361**	**0.210286863572**	**1.0**
04D3ks	**0.752**	**43.3018298804**	**0.207071436502**	**0.999999999949**
04D3hn	**0.5516**	**42.3031837333**	**0.149254872779**	**0.0**
04D3fk	**0.3578**	**41.4349095934**	**0.136096151868**	**0.0027167408**
04D3dd	**1.01**	**44.0124745362**	**0.375672417143**	**0.999785771595**
04D2ja	**0.741**	**43.7172522874**	**0.223286859678**	**0.0**
04D2gb	**0.43**	**41.8109207973**	**0.15108214332**	**0.001759266231**
04D1ak	**0.526**	**42.4002106795**	**0.16694309723**	**0.551671793295**
03D4gg	**0.592**	**42.5773227618**	**0.172394047168**	**0.0**
03D4di	**0.905**	**43.6387400062**	**0.25746754741**	**0.860692933969**
03D4cx	**0.949**	**43.4449098532**	**0.284630783131**	**9.00002294912-12**
03D3cd	**0.4607**	**42.0716665463**	**0.18126245549**	**0.999972076003**
03D3ay	**0.3709**	**41.6692043983**	**0.154834006045**	**0.000892345443**
03D1fq	**0.8**	**43.7140601053**	**0.215165244047**	**5.37990000327-08**
03D1co	**0.679**	**43.4574063402**	**0.195969077309**	**0.999996816858**
03D1aw	**0.5817**	**42.6720406074**	**0.159803369147**	**1.0**
04D4bq	**0.55**	**42.2756470464**	**0.15520464155**	**0.916905651559**
04D3ny	**0.81**	**43.3698924946**	**0.212685692189**	**0.399751945497**
04D3ml	**0.95**	**43.9766075066**	**0.291243768847**	**0.620797641982**
04D3kr	**0.3373**	**41.2944909521**	**0.137240588972**	**1.0**
04D3gx	**0.91**	**44.305516423**	**0.267241049423**	**0.551671793295**
04D3ez	**0.263**	**40.6346721186**	**0.134567832845**	**0.0**
04D3cy	**0.643**	**43.0112669877**	**0.166369771753**	**0.474555440866**
04D2iu	**0.691**	**43.0887669301**	**0.253842244922**	**0.551671793295**
04D2fs	**0.357**	**41.4255279641**	**0.137721476533**	**0.99999988436**
04D1aj	**0.721**	**43.1756783825**	**0.187179852377**	**0.551671793295**
03D4gf	**0.581**	**42.7423233406**	**0.155462517843**	**1.0**
03D4dh	**0.6268**	**42.7586126154**	**0.157227804978**	**0.999411415446**
03D4cn	**0.818**	**43.3928941721**	**0.26755880574**	**0.551671793295**
03D3cc	**0.4627**	**42.0422849962**	**0.14756880162**	**0.551671793295**
03D3aw	**0.449**	**42.0230683496**	**0.157063020734**	**0.0**
03D1fl	**0.688**	**43.0539860347**	**0.165777265258**	**0.551671793295**
03D1cm	**0.87**	**44.2313042047**	**0.278817290376**	**0.551671793295**

03D1au	0.5043	42.3382006953	0.149577507475	0.999977235075
b010	0.591	43.2091059677	0.306006112419	0.551671793295
b013	0.426	41.7615774796	0.22492067105	0.551671793295
b016	0.329	41.3402567282	0.269807401411	0.551671793295
d058	0.583	42.4011253659	0.28648393048	0.551671793295
d084	0.519	43.071509714	0.314462491184	0.551671793295
d085	0.401	41.6954244654	0.229308722315	0.551671793295
d086	0.205	39.9037694347	0.217272003785	0.551671793295
d087	0.34	41.2131787728	0.228190770173	0.551671793295
d089	0.436	41.8800161997	0.21653394713	0.551671793295
d093	0.363	41.551975686	0.208173297091	0.551671793295
d097	0.436	41.9257693002	0.228262759927	0.551671793295
d117	0.309	41.1737957091	0.235823875951	0.551671793295
d149	0.342	41.3845981768	0.21481032067	0.551671793295
e020	0.159	39.4163640953	0.242628007812	0.551671793295
e029	0.332	41.2554783013	0.239903276269	0.551671793295
e108	0.469	42.3544812112	0.28315240581	0.551671793295
e132	0.239	40.2591287799	0.204167929675	0.551671793295
e136	0.352	41.423779352	0.224191540842	0.551671793295
e138	0.612	42.813940944	0.283023414891	0.551671793295
e140	0.631	42.3773831209	0.244974123167	0.551671793295
e147	0.645	42.8169731662	0.234273129747	0.551671793295
e148	0.429	41.8900388123	0.215471389689	0.551671793295
e149	0.497	42.0757738154	0.221637712856	0.551671793295
f011	0.539	42.2266632084	0.241797816352	0.551671793295
f041	0.561	42.8719504017	0.306521908685	0.551671793295
f076	0.41	41.3474933283	0.256981142148	0.551671793295
f096	0.412	41.4237121884	0.312056381676	0.551671793295
f216	0.599	42.7434648961	0.382165112455	0.551671793295
f231	0.619	43.056018637	0.247822481956	0.551671793295
f235	0.422	41.7283094576	0.230713845858	0.551671793295
f244	0.54	42.5111635186	0.282945441834	0.551671793295
f308	0.401	42.5546203602	0.369893982362	0.551671793295
g005	0.218	40.0754074212	0.222222737823	0.551671793295
g050	0.633	42.2016724827	0.259877853306	0.551671793295
g052	0.383	41.6420339038	0.251010174803	0.551671793295
g055	0.302	41.3107982848	0.272346550274	0.551671793295
g097	0.34	41.0800432546	0.231193988299	0.551671793295
g120	0.51	41.8806853688	0.226762461825	0.551671793295
g133	0.421	42.1360316298	0.322004955245	0.551671793295
g142	0.399	41.4880817606	0.292952212036	0.551671793295
g160	0.493	42.1469146695	0.2639575465	0.551671793295
g240	0.687	42.9963850785	0.285188798534	0.551671793295
h300	0.687	42.8348351217	0.272537782393	0.551671793295
h319	0.495	42.2486633669	0.232347965963	0.551671793295
h323	0.603	42.6462047067	0.278628902676	0.551671793295
h342	0.421	42.1796277538	0.233773507359	0.551671793295
h359	0.348	41.5896262003	0.217425302485	0.551671793295
h363	0.213	40.109041203	0.22335261095	0.551671793295
h364	0.344	41.1728711166	0.209772839036	0.551671793295
k396	0.271	40.5323020705	0.21526700069	0.551671793295
k411	0.564	42.3728917486	0.292022400672	0.551671793295
k425	0.274	40.7246506476	0.208670920573	0.551671793295
k429	0.181	39.6828885172	0.227088514354	0.551671793295
k430	0.582	43.1639102562	0.325207917744	0.551671793295
k441	0.68	42.9040344577	0.290951926367	0.551671793295
k448	0.401	41.9356323131	0.332838743195	0.551671793295
k485	0.416	41.5575369467	0.303252121943	0.551671793295
m027	0.286	41.2111308437	0.25814462102	0.551671793295
m034	0.562	43.0505225945	0.33321544156	0.551671793295
m043	0.266	40.3907073912	0.250227167194	0.551671793295
m062	0.314	41.2319680355	0.242363893683	0.551671793295
m138	0.581	43.668679767	0.391236930244	0.551671793295
m158	0.463	41.947753754	0.26646445376	0.551671793295
m193	0.341	40.9915138593	0.224504319999	0.551671793295
n256	0.631	42.8818252933	0.234369669625	0.551671793295
n258	0.522	42.6773303466	0.24715086884	0.551671793295
n263	0.368	41.4676952372	0.207180966902	0.551671793295
n278	0.309	40.8607746451	0.214863256477	0.551671793295
n285	0.528	42.3696918414	0.302654151443	0.551671793295
n404	0.216	40.4049089433	0.21521177477	0.551671793295
p455	0.284	40.8353825838	0.206616021304	0.551671793295
p524	0.508	42.1942707319	0.231767202806	0.551671793295
p528	0.781	43.4372295515	0.321348684213	0.551671793295
p534	0.613	42.6167095942	0.260214469673	0.551671793295

1999fw	0.278	40.5662405695	0.198452987813	0.551671793295
1999fn	0.477	42.0543351554	0.183599933891	0.551671793295
1999fm	0.95	43.6383176075	0.297754710705	0.551671793295
1999fk	1.057	44.1506301001	0.238584685235	0.551671793295
1999fj	0.816	43.6910828816	0.457863421823	0.551671793295
1999ff	0.455	42.3239322721	0.229627540013	0.551671793295
04Eag	1.02	44.3629247027	0.220963762065	0.551671793295
04Gra	1.14	44.322957093	0.228701029542	0.551671793295
04Man	0.854	43.6084642861	0.225569905838	0.551671793295
04Mcg	1.37	45.049713349	0.262748449529	0.551671793295
04Omb	0.975	44.3337254974	0.21278060312	0.551671793295
04Pat	0.97	44.4626016974	0.291454456047	0.551671793295
04Rak	0.74	43.2875467038	0.198402487067	0.551671793295
04Sas	1.39	44.8762343403	0.250631965185	0.551671793295
04Yow	0.46	42.1484496113	0.210095497723	0.551671793295
05Fer	1.02	44.1640655276	0.232339527378	0.551671793295
05Gab	1.12	44.5144054871	0.224079094547	0.551671793295
05Lan	1.23	45.0206757228	0.235028400564	0.551671793295
05Red	1.19	44.3628316562	0.247108113742	0.551671793295
05Spo	0.839	43.3980657237	0.222124147444	0.551671793295
05Str	1.01	44.9121378293	0.33374861699	0.551671793295
05Zwi	0.521	42.3782445565	0.199332155448	0.551671793295
2002dc	0.475	42.1048814028	0.255958316646	0.551671793295
2002dd	0.95	43.884150294	0.235845124417	0.551671793295
2002fw	1.3	45.0162581016	0.242719167821	0.551671793295
2002hp	1.305	44.740169328	0.25954011501	0.551671793295
2002kc	0.216	40.5560465985	0.244099860655	0.551671793295
2002kd	0.735	43.0918403533	0.200575809439	0.551671793295
2002ki	1.14	44.1969521333	0.368403293803	0.551671793295
2003aj	1.307	45.4107441086	0.314696709568	0.551671793295
2003az	1.265	44.944110841	0.235469959716	0.551671793295
2003bd	0.67	43.1432095538	0.209654285426	0.551671793295
2003be	0.64	42.9242754337	0.280114332269	0.551671793295
2003dy	1.34	45.0675055813	0.275015093127	0.551671793295
2003eq	0.84	43.5143043773	0.208697053243	0.551671793295
2003XX	0.935	43.5401972359	0.227616538358	0.551671793295
2001cw	0.953	44.2736202267	0.954717441018	0.551671793295
2001gn	1.124	44.5675191814	0.197263047621	0.551671793295
2001go	0.552	42.5109342761	0.103414502954	0.551671793295
2001gq	0.671	42.9820070011	0.12054552758	0.551671793295
2001gy	0.511	42.3736604669	0.0890617545983	0.551671793295
2001hb	1.03	44.2400933572	0.141254202601	0.551671793295
A-004	1.192	44.4587516	0.200258071902	0.551671793295
C-000	1.092	44.0078477189	0.246511864633	0.551671793295
C-001	0.974	43.8341432564	0.174238940337	0.551671793295
F-012	1.11	44.6253358523	0.447832632795	0.551671793295
G-00	1.35	44.8270654771	0.185705412075	0.551671793295
H-00	0.85	43.4942573455	0.171796430655	0.551671793295
H-005	1.241	44.5817029608	0.478371053196	0.0
K-000	1.414	44.8037661387	0.346181483252	0.0
N-033	1.188	44.6076425498	0.500544949291	0.551671793295
D-000	1.017	44.2939707709	0.171046154322	1.4752251-05
D-006	1.315	44.9713577744	0.187508839367	0.551671793295
P-009	0.821	43.6409387868	0.194013591972	0.551671793295
R-012	1.215	45.2465209495	0.560317445151	0.000147652666
Z-005	0.623	42.5145239973	0.241428134977	0.551671793295

8.2. Annex B: MatLab Code

The code was programmed by Catarina Rocha, Mar Pino and Maximilian von Wietersheim with the guidance of Prof. Dr. Carlos Martins, and it follows this diagram:



```
load data.dat
z=data(:,2); % here we load the redshifts
d=data(:,3); % here we load the distance modulus
errord=data(:,4);

figure(1) %raw data plot
errorbar(z,d,errord,'*r');
xlabel('Redshift')
ylabel('Distance Modulus')
print -dpng distance_plot


c=299792.458;
a=(1+z).^2-1;
b=(1+z).^2+1;
vrel=c*(a./b); % convert redshift to recession velocity (relativistic way)
v=c*z; % convert redshift to recession velocity (Doppler shift approx)

Dl=10.^((d-25)/5); %convert distance modulus into luminosity distance

errorDl=(log(10)/5)*errord.*Dl;

% vrelp1=23.79; %
% vrelp2=2.584e+04;
% vp1=41.15;
% vp2=1.749e+04;
%
% Dlfit=0:10^3:2.5*10^5;
% vrelfit=vrelp1*Dlfit+vrelp2;
% Dlfit=0:10^3:2.5*10^5;
% vfit=vp1*Dlfit+vp2;
%
figure(2) %comparison of relativistically and non-rel. calculated data
hold on
% errorbar(Dl,vrel,'+b');
% errorbar(Dl,v,'+r');
plot(Dl,v,'k.',Dl,vrel,'r.')
ylabel('Recession Velocity (km/s)')
xlabel('Luminosity Distance (Mpc)')
legend('Doppler shift approx.','Relativistically calculated')
hold off
print -dpng velocity_compared


figure(3) %slope=H0 over the whole range of measurements
errorbar(vrel,Dl,errorDl,'+b');
xlabel('Recession Velocity (km/s)')
ylabel('Luminosity Distance (Mpc)')
print -dpng hubble_plot


p1=0.05492;  %values must be changed when data set is changed
p1max=0.05737;
p1min=0.05247;
p2= -2739;
Rsquare=0.7702;

H0=1/p1; %hubble parameter
errorH0=((p1max-p1min)/2)/(p1^2);
```

```
lowz=z(z<0.03);
lowa=(1+lowz).^2-1;
lowb=(1+lowz).^2+1;
lowv=c*(lowa./lowb); % convert redshift to recession velocity (relativity),
with LOW redshifts
lowd=d(z<0.03);
errorlowd=errord(z<0.03);
lowDl=10.^((lowd-25)/5);
errorlowDl=(log(10)/5)*errorlowd.*lowDl;

figure(4)
errorbar(lowv,lowDl,errorlowDl,'+g'); %slope=H0 from the most recent data
xlabel('Restricted Recession Velocity (km/s)')
ylabel('Restricted Luminosity Distance (Mpc)')
print -dpng restricted_hubble_plot


lowp1= 0.01662; %values must be changed when data set is changed
lowp1max=0.01832;
lowp1min=0.01492;
lowp2=-9.579;
lowRsquare=0.8183;

xfit1=0:10^3:2.5*10^5;
yfit1=p1*xfit1+p2;
xfit2=0:10^3:2.5*10^5;
yfit2=lowp1*xfit2+lowp2;

restrictedH0=1/lowp1; %"today's" hubble parameter
error_restrictedH0=((lowp1max-lowp1min)/2)/(lowp1^2);

figure(5) %comparison of H0 using recent data and using all data
hold on
errorbar(vrel,Dl,errorDl,'+b');
errorbar(lowv,lowDl,errorlowDl,'+g');
plot(xfit1,yfit1,'k--',xfit2,yfit2,'k-')
xlabel('Recession Velocity (km/s)')
ylabel('Luminosity Distance (Mpc)')
legend('All data','Low redshift data (z<0.03)','All fit','Low redshift fit
(z<0.03)')
hold off
print -dpng combined_hubble_plot


omm=1; %z vs. Dl if matter dominated universe
omr=0;
H0=70;
zmax=1.414;
N=580;

zgrid=0:zmax/579:zmax;
Int=zeros(1,N);
for i=2:N;
    Int(i)=Int(i-1)+(zmax/N)/sqrt(omm*(1+zgrid(i)).^3);
end

Dlomm1=(c/H0)*(1+zgrid).*Int;

omm027=0.27; %z vs. Dl for assummed omm and omr
```

```
omr073=0.73;

zgrid=0:zmax/579:zmax;
Int=zeros(1,N);
for i=2:N;
    Int(i)=Int(i-1)+(zmax/N)/sqrt(omm027*(1+zgrid(i)).^3+omr073);
end

Dlomm027=(c/H0)*(1+zgrid).*Int;

figure(6) %plot of model lines and data
hold on
plot(zgrid,Dlomm1,'-b');
plot(zgrid,Dlomm027,'+r');
errorbar(z,Dl,errorDl,'*g')
xlabel('Redshift')
ylabel('Luminosity Distance (Mpc)')


hubble=@(z,omm,omr)1./sqrt(omm*(1+z).^3+omr);

zmax=1.414;
H0=70;

zbin=0:0.1:1.5;

for i=1:16
    Dlt(i)=c/H0*(1+zbin(i))*integral(@(z)hubble(z,0.27,0.73),0,zbin(i));
end
plot(zbin,Dlt,'-k') % data best fit line (overlaping with assummed model)
legend('\Omega_M=1','\Omega_M=0.27','Supernovae Data', 'Data fit')

hold off
print -dpng z_Dlommx_compared


H0=72; %1D likelihood (loglikelihood) for omega mass using supernovae data
(fixed H0)
omm=0.0:1/100:1;
omr=1-omm;
Loglike=zeros(101:1);
for j=1:101
    Loglike(j)=0;
    for i=1:580
        zgrid=0:z(i)/100:z(i);
        Int=0;
        for k=1:101
            Int=Int+(z(i)/100)/sqrt(omm(j)*(1+zgrid(k)).^3+omr(j));
        end
        Dlt=(c/H0)*(1+z(i))*Int;
        Loglike(j)=Loglike(j)-(1/2)*(Dlt-Dl(i)).^2/errorDl(i).^2;
    end
end

omm=0.0:1/100:1; %1D likelihood (loglikelihood) for omega mass using
supernovae data (ranged H0)
omr=1-omm;
Loglikem=zeros(101:1);
for j=1:101
```

```
    Loglikem(j)=0;
    for l=1:5
        H0=69+l;
    for i=1:580
        zgrid=0:z(i)/100:z(i);
        Int=0;
        for k=1:101
            Int=Int+(z(i)/100)/sqrt(omm(j)*(1+zgrid(k)).^3+omr(j));
        end
        Dlt=(c/H0)*(1+z(i))*Int;
        Loglikem(j)=Loglikem(j)-(1/2)*(Dlt-Dl(i)).^2/errorDl(i).^2;
    end
    Loglikem(j)=Loglikem(j)/5
    end
end

figure(7)
hold on
plot(omm,Loglike,'b-',omm,Loglikem,'r--');
ylabel('Loglike')
xlabel('\Omega_M')
legend('H_0=72','H_0(marginalized)')
print -dpng ommass_vs_loglike


chisq=-2*Loglike; %1D likelihood (chi-squared) for omega mass using
supernovae data
chisq=chisq-min(chisq);
chisqm=-2*Loglikem;
chisqm=chisqm-min(chisqm);

figure(8)
hold on
plot(omm,chisq,'b-',omm,chisqm,'r--');
ylabel('\chi^2')
xlabel('\Omega_M')
legend('H_0=72','H_0(marginalized)')
print -dpng ommass_vs_chisq


xtemp=0.16:0.0001:0.39;

figure(9)%same as figure 8, but with a shorter range of values/ zooming in
hold on
plot(omm,chisq,'b-',omm,chisqm,'r--'); %changed range because minimum
plot(xtemp,1,'k-');
plot(xtemp,4,'k-');
plot(xtemp,9,'k-');
axis([0.16 0.39 0 10])
ylabel('\chi^2')
xlabel('\Omega_M')
legend('H_0=72','H_0(marginalized)')
hold off
print -dpng ommassranged_vs_chisq


H0=72; %opacity test (long process)
N=100;
omm=0:1/N:1;
```

```
omr=1-omm;
eps=-0.5:1/N:0.5;
Loglike=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike(j,l)=0;
        for i=1:580
            zgrid=0:z(i)/100:z(i);
            Int=0;
            for k=1:101
                Int=Int+(z(i)/100)/sqrt(omm(j)*(1+zgrid(k)).^3+omr(j));
            end
            Dlt=(c/H0)*(1+z(i))*Int;
            Loglike(j,l)=Loglike(j,l)-(1/2)*(Dlt*exp(eps(l)*z(i))-
Dl(i)).^2/errorDl(i).^2;
        end
    end
end

figure(10)
contour(eps,omm,Loglike)
contour(eps,omm,Loglike,[max(max(Loglike))-5.9,max(max(Loglike))-
3.1,max(max(Loglike))-1.15])
ylabel('\Omega_M')
xlabel('\epsilon')
zlabel('Loglike')
print -dpng opacity_test


zlittle=z(z<0.5); %restricted version of the above opacity test (fig 10)
Dllittle=Dl(z<0.5);
errorDllittle=errorDl(z<0.5);
H0=72;
N=100;
omm=0:1/N:1;
omr=1-omm;
eps=-0.5:1/N:0.5;
Loglike=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike(j,l)=0;
        for i=1:412
            zgrid=0:z(i)/100:z(i);
            Int=0;
            for k=1:101
                Int=Int+(z(i)/100)/sqrt(omm(j)*(1+zgrid(k)).^3+omr(j));
            end
            Dlt=(c/H0)*(1+z(i))*Int;
            Loglike(j,l)=Loglike(j,l)-(1/2)*(Dlt*exp(eps(l)*z(i))-
Dl(i)).^2/errorDl(i).^2;
        end
    end
end

figure(11)
contour(eps,omm,Loglike)
contour(eps,omm,Loglike,[max(max(Loglike))-5.9,max(max(Loglike))-
3.1,max(max(Loglike))-1.15])
ylabel('\Omega_M')
xlabel('\epsilon')
```

```
zlabel('Loglike')
print -dpng opacity_test_restricted


omm=0.27; %equation of state: 1D (w0) constant prior array (assuming flat
universe)
omd=1-omm;
N=100;
w0=-2:2/N:0;
Loglike=zeros(N+1:1);
H0=72;
for j=1:101
    Loglike(j)=0;
    for i=1:580;
        zgrid=0:z(i)/100:z(i);
        Int=0;
        for k=1:N+1

Int=Int+(z(i)/100)/(sqrt(omm*(1+zgrid(k)).^3+omd*(1+zgrid(k)).^(3*(1+w0(j))
)));
        end
        Dlt=(c/H0)*(1+z(i))*Int;
        Loglike(j)=Loglike(j)-(1/2)*(Dlt-Dl(i)).^2/errorDl(i).^2;
    end
end

Logliken=zeros(101:1);%the same as the above (fig 12), but running over
range of omm values
w0=-2:2/N:0;
H0=72;
for j=1:101
    Logliken(j)=0;
    for i=1:580
        zgrid=0:z(i)/100:z(i);
        for l=1:13
            omm=0.245+0.005*l;
            omd=1-omm;
            Int=0;
            for k=1:101

Int=Int+(z(i)/100)./(sqrt(omm*(1+zgrid(k)).^3+omd*(1+zgrid(k)).^(3.*(1+w0(j
)))));
            end
        Dlt=(c/H0)*(1+z(i))*Int;
        Logliken(j)=Logliken(j)-(1/2)*(Dlt-Dl(i)).^2/errorDl(i).^2;
        end
    end
end

Logliken=Logliken/13;

xtemp=-2.5:0.0001:0.5;

figure(12)
hold on
plot(w0,-2*Loglike-min(-2*Loglike),'b-')
plot(w0,-2*Logliken-min(-2*Logliken),'r-');
legend('\Omega_M=0.27','\Omega_M (marginalized)')
plot(xtemp,1,'k-');
plot(xtemp,4,'k-');
```

```
plot(xtemp,9,'k-');
axis([-2 0 0 10])
ylabel('\chi^2')
xlabel('w_0')
hold off
print -dpng w0_vs_loglikelihood


H0=72; %equation of state: 2D (w0,omm) constant prior array (assuming flat
universe)
N=100;
omm=0:1/N:1;
omd=1-omm;
w0=-2:2/N:0
Loglike=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike(j,l)=0;
        for i=1:580
            zgrid=0:z(i)/100:z(i);
            Int=0;
            for k=1:101

Int=Int+(z(i)/100)/(sqrt(omm(j).*(1+zgrid(k)).^3+omd(j).*(1+zgrid(k)).^(3*(
1+w0(l)))));
            end
            Dlt=(c/H0)*(1+z(i))*Int;
            Loglike(j,l)=Loglike(j,l)-(1/2).*(Dlt-Dl(i)).^2./errorDl(i).^2;
        end
    end
end

figure(13)
hold on
contour(w0,omm,Loglike,[max(max(Loglike))-5.9,max(max(Loglike))-
3.1,max(max(Loglike))-1.15])
ylabel('\Omega_M')
xlabel('w_0')
zlabel('Loglike')

H0=70; %the same as the above (fig 13) with different H0 prior
N=100;
omm_1=0:1/N:1;
omd_1=1-omm_1;
w0_1=-2:2/N:0
Loglike_1=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike_1(j,l)=0;
        for i=1:580
            zgrid=0:z(i)/100:z(i);
            Int=0;
            for k=1:101

Int=Int+(z(i)/100)/(sqrt(omm_1(j).*(1+zgrid(k)).^3+omd_1(j).*(1+zgrid(k)).^
(3*(1+w0_1(l)))));
            end
            Dlt=(c/H0)*(1+z(i))*Int;
            Loglike_1(j,l)=Loglike_1(j,l)-(1/2).*(Dlt-
Dl(i)).^2./errorDl(i).^2;
```

```
        end
    end
end

contour(w0_1,omm_1,Loglike_1,[max(max(Loglike_1))-5.9,max(max(Loglike_1))-
3.1,max(max(Loglike_1))-1.15])
hold off
print -dpng w0_vs_omm

load hubble.dat
zh=hubble(:,1);
Hz=hubble(:,2);
errorHz=hubble(:,3);
c=299792.458;

N=100;
H0=68;
omm=0:1/N:1;
omd=1-omm;
w0=-2:2/N:0;
Loglike=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike(j,l)=0;
        for i=1:28
            
Hzt=H0*sqrt(omm(j)*(1+zh(i)).^3+omd(j)*(1+zh(i)).^(3*(1+w0(l))));
            Loglike(j,l)=Loglike(j,l)-(1/2).*(Hz(i)-Hzt).^2./errorHz(i).^2;
        end
    end
end

figure(14)
hold on
contour(w0,omm,Loglike,[max(max(Loglike))-5.9,max(max(Loglike))-
3.1,max(max(Loglike))-1.15])
ylabel('\Omega_M')
xlabel('w_0')
zlabel('Loglike')

Transposedm=Loglike';

figure(15)
hold on
contour(w0,omm,Transposedm,[max(max(Loglike))-5.9,max(max(Loglike))-
3.1,max(max(Loglike))-1.15])
xlabel('\Omega_M')
ylabel('w_0')
zlabel('Loglike')


N=100;
H0=73.8;
omm_1=0:1/N:1;
omd_1=1-omm;
w0_1=-2:2/N:0;
Loglike_1=zeros(N+1:N+1);
for j=1:N+1
    for l=1:N+1
        Loglike_1(j,l)=0;
```

```
        for i=1:28
            zgrid=0:z(i)/100:z(i);
            Int=0;
            Hzt=H0*Int;
            Loglike_1(j,l)=Loglike_1(j,l)-(1/2).*(Hz(i)-
Hzt(i,j,l)).^2./errorHz(i).^2;
        end
    end
end

contour(w0_1,omm_1,Loglike_1,[max(max(Loglike_1))-5.9,max(max(Loglike_1))-
3.1,max(max(Loglike_1))-1.15])
hold off
print -dpng w0_vs_omm
```