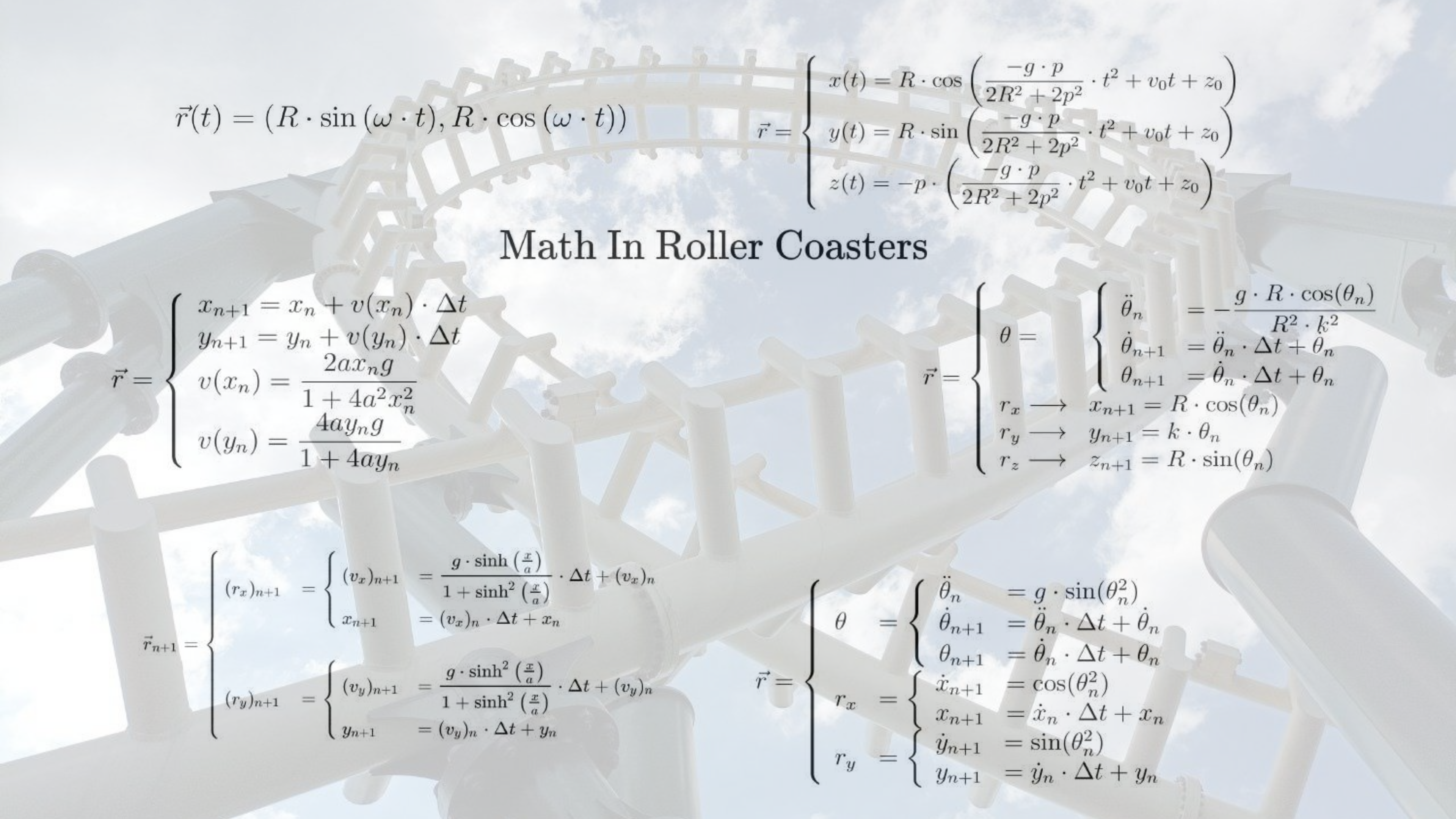

# Math In Roller Coasters

$$\vec{r}(t) = (R \cdot \sin(\omega \cdot t), R \cdot \cos(\omega \cdot t))$$

$$\vec{r} = \begin{cases} x(t) = R \cdot \cos\left(\dfrac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \\ y(t) = R \cdot \sin\left(\dfrac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \\ z(t) = -p \cdot \left(\dfrac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \end{cases}$$

$$\vec{r} = \begin{cases} x_{n+1} = x_n + v(x_n) \cdot \Delta t \\ y_{n+1} = y_n + v(y_n) \cdot \Delta t \\ v(x_n) = \dfrac{2ax_n g}{1 + 4a^2 x_n^2} \\ v(y_n) = \dfrac{4ay_n g}{1 + 4ay_n} \end{cases}$$

$$\vec{r} = \begin{cases} \theta = \begin{cases} \ddot{\theta}_n & = -\dfrac{g \cdot R \cdot \cos(\theta_n)}{R^2 \cdot k^2} \\ \dot{\theta}_{n+1} & = \ddot{\theta}_n \cdot \Delta t + \theta_n \\ \theta_{n+1} & = \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x \longrightarrow \quad x_{n+1} = R \cdot \cos(\theta_n) \\ r_y \longrightarrow \quad y_{n+1} = k \cdot \theta_n \\ r_z \longrightarrow \quad z_{n+1} = R \cdot \sin(\theta_n) \end{cases}$$

$$\vec{r}_{n+1} = \begin{cases} (r_x)_{n+1} & = \begin{cases} (v_x)_{n+1} & = \dfrac{g \cdot \sinh\left(\frac{x}{a}\right)}{1 + \sinh^2\left(\frac{x}{a}\right)} \cdot \Delta t + (v_x)_n \\ x_{n+1} & = (v_x)_n \cdot \Delta t + x_n \end{cases} \\ (r_y)_{n+1} & = \begin{cases} (v_y)_{n+1} & = \dfrac{g \cdot \sinh^2\left(\frac{x}{a}\right)}{1 + \sinh^2\left(\frac{x}{a}\right)} \cdot \Delta t + (v_y)_n \\ y_{n+1} & = (v_y)_n \cdot \Delta t + y_n \end{cases} \end{cases}$$

$$\vec{r} = \begin{cases} \theta & = \begin{cases} \ddot{\theta}_n & = g \cdot \sin(\theta_n^2) \\ \dot{\theta}_{n+1} & = \ddot{\theta}_n \cdot \Delta t + \dot{\theta}_n \\ \theta_{n+1} & = \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x & = \begin{cases} \dot{x}_{n+1} & = \cos(\theta_n^2) \\ x_{n+1} & = \dot{x}_n \cdot \Delta t + x_n \end{cases} \\ r_y & = \begin{cases} \dot{y}_{n+1} & = \sin(\theta_n^2) \\ y_{n+1} & = \dot{y}_n \cdot \Delta t + y_n \end{cases} \end{cases}$$

# Index

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- Vectorial projection with respect to a variable
- Vectorial projection with respect to a parameter
- Euler's method

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# Motivation

# Motivation and objective

Why math?

$$\sum F = m \cdot a$$

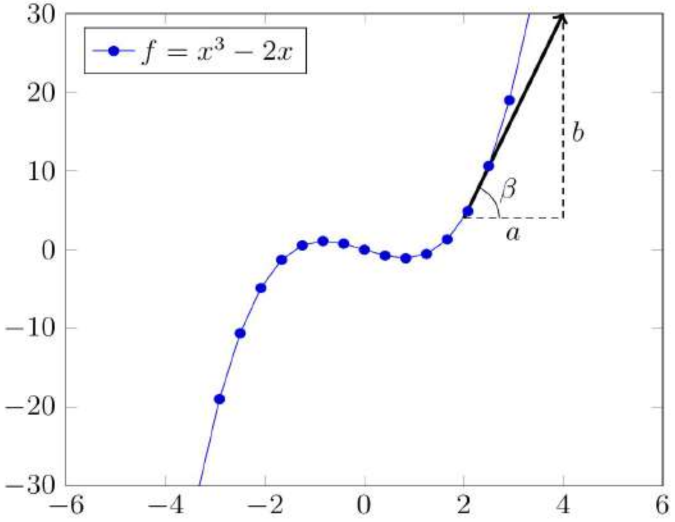
What is the objective?

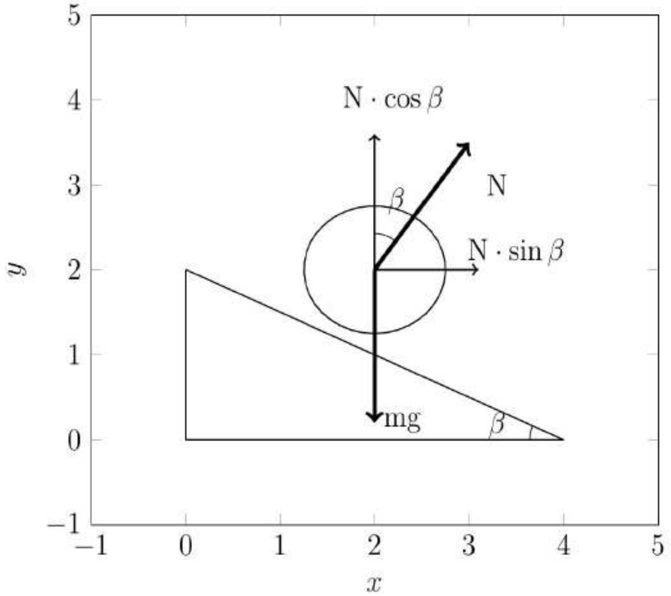
$$r(\theta) \longrightarrow r(t)$$

What is Blender3D ?

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH


# 1. Trigonometric Projection



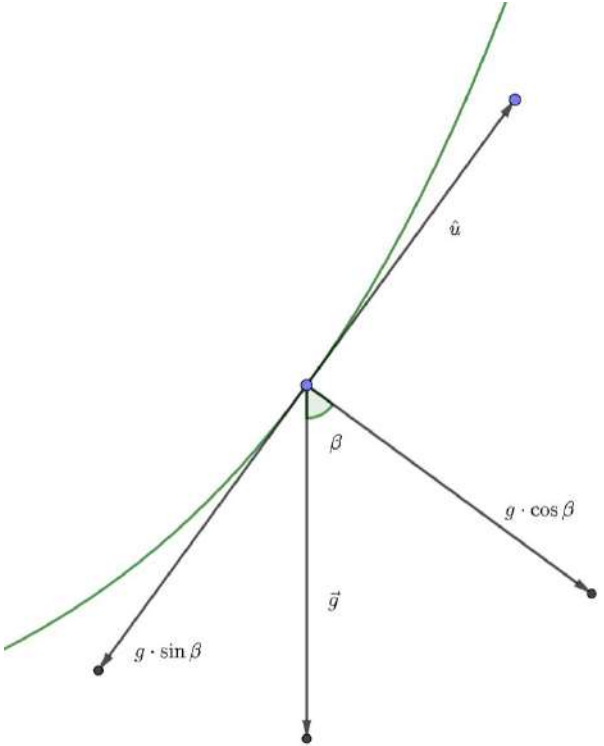

$$\|\vec{a}_T\| = g \cdot \cos\theta \qquad \|\vec{a}_Z\| = \|\vec{a}_T\| \cdot \cos\theta = g \cdot \cos^2\theta$$

# 2. Vectorial projection with respect to a variable

$$\vec{u} \cdot \vec{g} = \|\vec{u}\| \cdot \|\vec{g}\| \cdot \cos\sigma$$

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{(1, f'(x))}{\sqrt{1 + f'^2(x)}}$$

$$\vec{a} = \left( \frac{g \cdot f'(x)}{1 + f'^2(x)}, \frac{g \cdot f'^2(x)}{1 + f'^2(x)} \right) = (\vec{a}_x, \vec{a}_y)$$

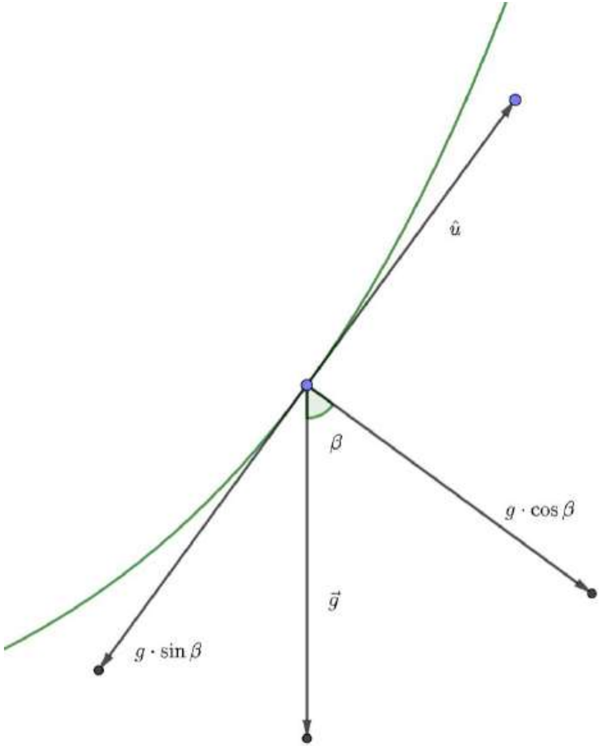


# 3. Vectorial projection with respect to a parameter

$$\vec{u} \cdot \vec{g} = \|\vec{u}\| \cdot \|\vec{g}\| \cdot \cos\sigma$$

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{(x'(\theta), y'(\theta), z'(\theta))}{\sqrt{x'^2(\theta) + y'^2(\theta) + z'^2(\theta)}}$$

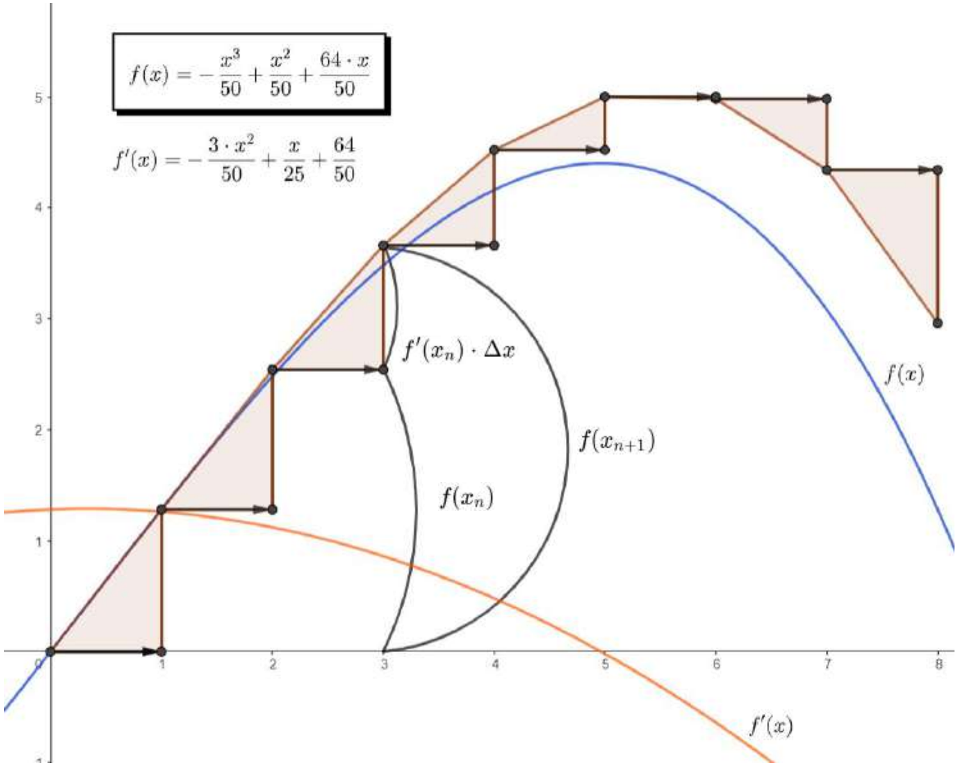
$$\vec{a} = \left(\frac{g \cdot z'(\theta) \cdot x'(\theta)}{x'^2(\theta) + y'^2(\theta) + z'^2(\theta)}, \frac{g \cdot z'(\theta) \cdot y'(\theta)}{x'^2(\theta) + y'^2(\theta) + z'^2(\theta)}, \frac{g \cdot z'^2(\theta)}{x'^2(\theta) + y'^2(\theta) + z'^2(\theta)}\right)$$



# 4. Euler’s method

$$f(x_{n+1}) = f'(x_n) \cdot \Delta x + f(x_n)$$

$$\vec{r} = \begin{cases} x_{n+1} = v_n \cdot \Delta t + x_n \\ v_{n+1} = a_n \cdot \Delta t + v_n \end{cases}$$



# Elements

1. Curve
2. Parabola
3. Catenary

Explicit elements

4. Helix
5. Vertical Loop
6. In-line twist
7. Corkscrew

Parameterized elements

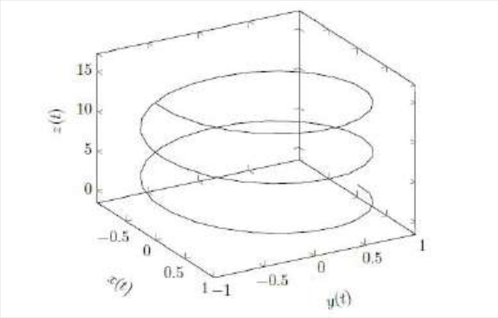
# Helix

$$\vec{r}(\theta) = \begin{cases} x(\theta) = R \cdot \cos(\theta) \\ y(\theta) = R \cdot \sin(\theta) \\ z(\theta) = -p \cdot \theta \end{cases}$$

$$-p \cdot \ddot{\theta}(t) = g \cdot \cos^2(\beta)$$

$$\theta(t) = \frac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0$$

$$\vec{r} = \begin{cases} x(t) = R \cdot \cos\left(\frac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \\ y(t) = R \cdot \sin\left(\frac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \\ z(t) = -p \cdot \left(\frac{-g \cdot p}{2R^2 + 2p^2} \cdot t^2 + v_0 t + z_0\right) \end{cases}$$




- [x] Tangential projection
- [ ] Vectorial projection w/ variable
- [ ] Vectorial projection w/ parameter
- [ ] Euler’s method

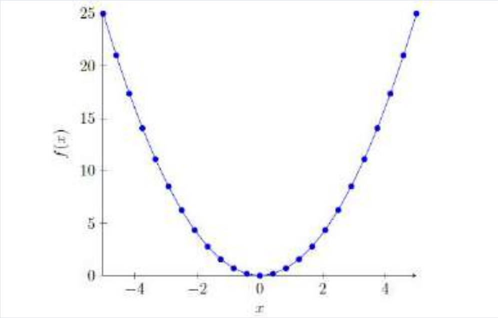
Difficulty: ★★★★☆

# Parabola (Camelback)

$$y = ax^2 + bx + c$$

$$\ddot{r}(t) = \begin{cases} \ddot{x}(t) = \dfrac{2ax(t)g}{1 + 4a^2x^2(t)} \\ \ddot{y}(t) = \dfrac{4ay(t)g}{1 + 4ay(t)} \end{cases}$$

$$\vec{r}(t) = \begin{cases} x_{n+1} = x_n + v(x_n) \cdot \Delta t \\ y_{n+1} = y_n + v(y_n) \cdot \Delta t \\ v(x_n) = \dfrac{2ax_ng}{1 + 4a^2x_n^2} \\ v(y_n) = \dfrac{4ay_ng}{1 + 4ay_n} \end{cases}$$




- ☒ Tangential projection
- ☑ Vectorial projection w/ variable
- ☐ Vectorial projection w/ parameter
- ☑ Euler’s method

Difficulty: ★★★★★

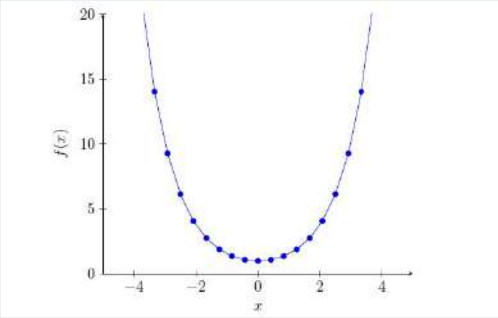
# Catenary

$$f(x) = a \cdot \cosh\left(\frac{x}{a}\right)$$

$$\vec{r}_x \quad = \begin{cases} \dot{v}_x & = \dfrac{g \cdot f'(x)}{1 + f'(x)} \\ \dot{x} & = v_x \end{cases}$$

$$\vec{r}_y \quad = \begin{cases} \dot{v}_y & = \dfrac{g \cdot f'(x)}{1 + f'(x)} \\ \dot{y} & = v_y \end{cases}$$

$$\vec{r}_{n+1} = \begin{cases} (r_x)_{n+1} & = \begin{cases} (v_x)_{n+1} & = \dfrac{g \cdot \sinh\left(\frac{x}{a}\right)}{1 + \sinh^2\left(\frac{x}{a}\right)} \cdot \Delta t + (v_x)_n \\ x_{n+1} & = (v_x)_n \cdot \Delta t + x_n \end{cases} \\ \\ (r_y)_{n+1} & = \begin{cases} (v_y)_{n+1} & = \dfrac{g \cdot \sinh^2\left(\frac{x}{a}\right)}{1 + \sinh^2\left(\frac{x}{a}\right)} \cdot \Delta t + (v_y)_n \\ y_{n+1} & = (v_y)_n \cdot \Delta t + y_n \end{cases} \end{cases}$$




- ☒ Tangential projection
- ☑ Vectorial projection w/ variable
- ☐ Vectorial projection w/ parameter
- ☑ Euler's method

Difficulty: ★☆☆☆☆

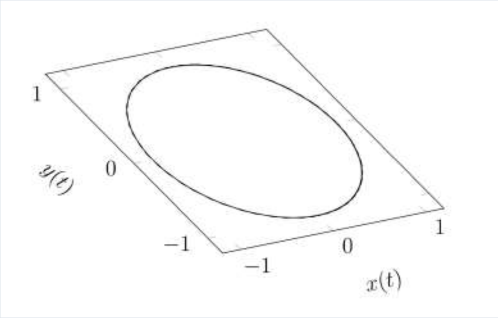
# Curve

$$x^2 + y^2 = R^2$$

$$\vec{r}(\varphi) = \begin{cases} x(\varphi) & = R \cdot \sin\varphi \\ y(\varphi) & = R \cdot \cos\varphi \end{cases}$$

$$\ddot{\varphi}(t) = 0$$

$$\vec{r}(t) = (R \cdot \sin(\omega \cdot t), R \cdot \cos(\omega \cdot t))$$




- [ ] Tangential projection
- [ ] Vectorial projection w/ variable
- [ ] Vectorial projection w/ parameter
- [ ] Euler's method

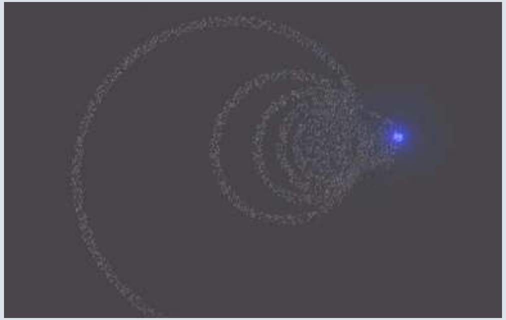
Difficulty: ★★☆☆☆

# Vertical Loop (Clothoid)

$$\vec{r}(\theta) = \begin{cases} C(\theta) & = \int\limits_0^\theta \cos\lambda^2 \, d\lambda \\ S(\theta) & = \int\limits_0^\theta \sin\lambda^2 \, d\lambda \end{cases}$$

$$\ddot{\theta} = g \cdot \sin(\theta)$$

$$\vec{r}(t) = \begin{cases} \theta & = \begin{cases} \ddot{\theta}_n & = g \cdot \sin(\theta_n^2) \\ \dot{\theta}_{n+1} & = \ddot{\theta}_n \cdot \Delta t + \dot{\theta}_n \\ \theta_{n+1} & = \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x & = \begin{cases} \dot{x}_{n+1} & = \cos(\theta_n^2) \\ x_{n+1} & = \dot{x}_n \cdot \Delta t + x_n \end{cases} \\ r_y & = \begin{cases} \dot{y}_{n+1} & = \sin(\theta_n^2) \\ y_{n+1} & = \dot{y}_n \cdot \Delta t + y_n \end{cases} \end{cases}$$



- ☒ Tangential projection
- ☒ Vectorial projection w/ variable
- ☑ Vectorial projection w/ parameter
- ☑ Euler's method

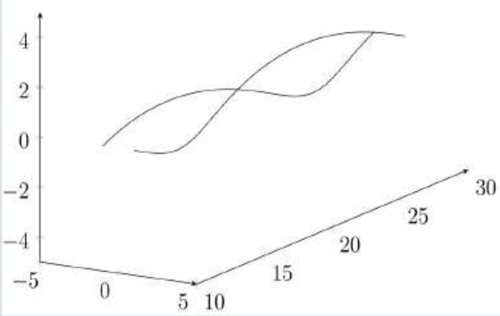
Difficulty: ★★★☆☆

# In-line Twist

$$\vec{r}(t) = \begin{cases} x(t) &= R \cdot \cos(\theta) \\ y(t) &= k \cdot \theta \\ z(t) &= R \cdot \sin(\theta) \end{cases}$$

$$\ddot{\theta} = \frac{g \cdot R \cdot \cos(\theta)}{R^2 + k^2}$$

$$\vec{r}(t) = \begin{cases} \theta = \begin{cases} \ddot{\theta}_n &= \dfrac{g \cdot R \cdot \cos(\theta_n)}{R^2 \cdot k^2} \\ \dot{\theta}_{n+1} &= \ddot{\theta}_n \cdot \Delta t + \dot{\theta}_n \\ \theta_{n+1} &= \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x \longrightarrow \quad x_{n+1} = R \cdot \cos(\theta_n) \\ r_y \longrightarrow \quad y_{n+1} = k \cdot \theta_n \\ r_z \longrightarrow \quad z_{n+1} = R \cdot \sin(\theta_n) \end{cases}$$



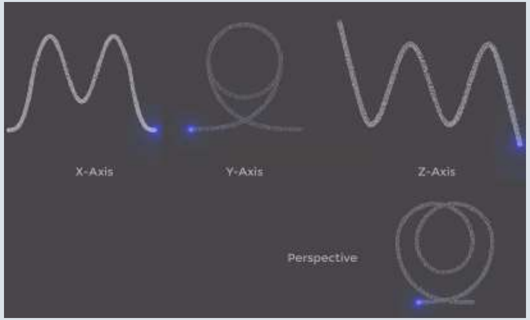

- ❌ Tangential projection
- ❌ Vectorial projection w/ variable
- ✅ Vectorial projection w/ parameter
- ✅ Euler's method

Difficulty: ★★★★★

# Corkscrew

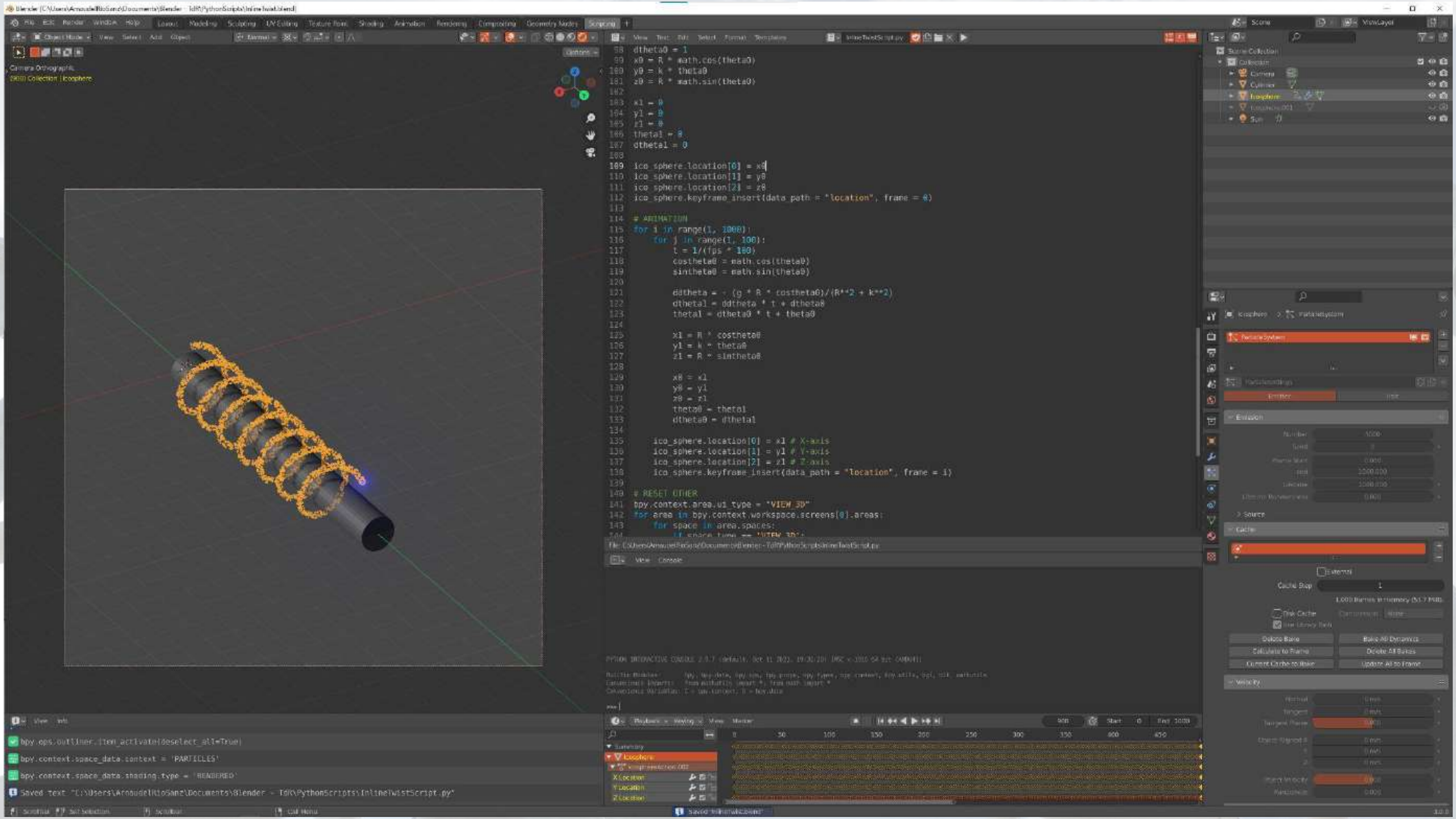
$$\vec{r}(t) = \begin{cases} \theta = \begin{cases} \ddot{\theta}_n & = \dfrac{g \cdot R \cdot \cos(\theta_n)}{R^2 \cdot k^2} \\ \dot{\theta}_{n+1} & = \ddot{\theta}_n \cdot \Delta t + \dot{\theta}_n \\ \theta_{n+1} & = \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x \longrightarrow \quad x_{n+1} = R \cdot \cos(\theta_n) \\ r_y \longrightarrow \quad y_{n+1} = k \cdot \theta_n \\ r_z \longrightarrow \quad z_{n+1} = R \cdot \sin(\theta_n) \end{cases}$$

$$\vec{r}(t) = \begin{cases} \theta = \begin{cases} \ddot{\theta}_n & = g \cdot \sin(\theta_n^2) \\ \dot{\theta}_{n+1} & = \ddot{\theta}_n \cdot \Delta t + \dot{\theta}_n \\ \theta_{n+1} & = \dot{\theta}_n \cdot \Delta t + \theta_n \end{cases} \\ r_x = \begin{cases} \dot{x}_{n+1} & = \cos(\theta_n^2) \\ x_{n+1} & = \dot{x}_n \cdot \Delta t + x_n \end{cases} \\ r_y = \begin{cases} \dot{y}_{n+1} & = \sin(\theta_n^2) \\ y_{n+1} & = \dot{y}_n \cdot \Delta t + y_n \end{cases} \end{cases}$$




- ✗ Tangential projection
- ✗ Vectorial projection w/ variable
- ✓ Vectorial projection w/ parameter
- ✓ Euler's method

Difficulty: ★★★☆☆


dtheta0 = 1
x0 = R * math.cos(theta0)
y0 = k * theta0
z0 = R * math.sin(theta0)
x1 = 0
y1 = 0
z1 = 0
theta1 = 0
dtheta1 = 0
ico_sphere.location[0] = x0
ico_sphere.location[1] = y0
ico_sphere.location[2] = z0
ico_sphere.keyframe_insert(data_path = "location", frame = 0)
# ANIMATION
for i in range(1, 1000):
for j in range(1, 100):
t = 1/(fps * 100)
costheta0 = math.cos(theta0)
sintheta0 = math.sin(theta0)
ddtheta = - (g * R * costheta0)/(R**2 + k**2)
dtheta1 = ddtheta * t + dtheta0
theta1 = dtheta0 * t + theta0
x1 = R * costheta0
y1 = k * theta0
z1 = R * sintheta0
x0 = x1
y0 = y1
z0 = z1
theta0 = theta1
dtheta0 = dtheta1
ico_sphere.location[0] = x1 # X-axis
ico_sphere.location[1] = y1 # Y-axis
ico_sphere.location[2] = z1 # Z-axis
ico_sphere.keyframe_insert(data_path = "location", frame = i)
# RESET OTHER
bpy.context.area.ui_type = "VIEW_3D"
for area in bpy.context.workspace.screens[0].areas:
for space in area.spaces:
bpy.ops.outliner.item_activate(deselect_all=True)
bpy.context.space_data.context = 'PARTICLES'
bpy.context.space_data.shading.type = 'RENDERED'

# VIDEO

# Conclusions

Difficult

Ingenuous

Complex

**Objective**

Fun

Interesting

Worthwhile

Good experience overall

Photo by Arnau del Río

# The End

# **Any questions?**

Thanks for listening